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**Diagnostic
Items for**
**Assessing
Mathematical
Understanding**

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2011

Portland, OR

**Diagnostic
Items for
Assessing
Mathematical
Understanding**



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Instructions

This assessment is one of four components of Assessing Mathematical Understanding. It is recommended that users familiarize themselves with the background, concept areas, learning goals, and organizational framework found in *A Guide for Assessing Mathematical Understanding* before using this assessment. Detailed instructions and sample records for using this assessment are found on pages 45–48 of the *Guide*. A blank student record for the diagnostic items can be found in the appendix of the *Guide*.

Preparation

1. **Collect the materials** necessary for the assessment.
2. **Set up a space** that is free from distractions and allows the teacher or other assessment administrator and student to sit comfortably face-to-face with the test booklet open on the table between them. There should be sufficient workspace for the student to lay out manipulatives and to write.
3. **Bring one student at a time** to the interview location.
4. **Read the introductory script.**
 - a. Say, “Today I am going to ask you some more number questions. I am really interested in how you think about these problems so I might ask you to explain your thinking.”
 - b. Say, “It’s OK to say, ‘I don’t know,’ or ‘Let’s move on,’ for any question.”
 - c. Say, “I will read a problem over again, if you ask me to.”
 - d. Say, “You may use any of the objects on the table to help you think about the question.”
 - e. Say, “Are you ready to begin? OK, let’s get started.” (Or wait, if the student has a question.)

Administration and Data Collection

5. **Read each item as printed and elaborate, if necessary.** The goal is for the student to be able to show what he or she knows.
 - a. You may paraphrase or repeat anything in the assessment.
 - b. You may offer manipulatives shown on each page.
 - c. Students may point (rather than speak) to indicate an answer when appropriate.
 - d. If a student does not know his or her colors or is unfamiliar with a vocabulary word, you may clarify.
 - e. There is no time limit for responses (except as indicated in the assessment).
 - f. Units are not required for correct answers. For example, “5” and “5 dogs” are both correct.
6. **Use the “For Further Diagnosis” suggestions** found on the teacher’s page for each item to gain additional information about a student’s thinking about that item. Additional probing questions and prompts that can be used include:
 - a. “How did you know that?”
 - b. “Tell me out loud what you did.”
 - c. “What were you thinking?”
 - d. “How did you figure that out?”
 - e. “Can you show me another way to do that?”

7. **Record detailed notes about student responses on the record.** The most important information to collect during the diagnostic assessment is detailed descriptions of the student response and strategies. If desired, the same codes from the grade-level assessment can be used to capture some information; however, no score will be computed for the diagnostic sections:

- Ⓒ correct answer given quickly with confidence
- C correct answer
- P partially correct answer
- X incorrect answer, no response to the problem, or “I don’t know”

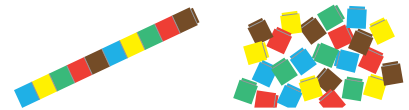
Other codes can be used to indicate details about student responses:

- ? indicates student hesitated or was puzzled by the task
- DM indicates student used direct modeling to solve (fingers, objects)
- CO indicates student used a counting strategy (counting on, counting back)
- NR indicates student used numerical reasoning to solve (known fact, mental calculation, number relationships)
- SC indicates student self-corrected (initially gave an incorrect answer, then changed to correct answer)

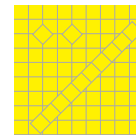
Advance Preparation

Materials needed

- A supply of about 60 linking cubes
 - 40 cubes arranged in sticks of 10 cubes each
 - 20 loose cubes



- A supply of base-ten blocks (5 flats, 15 longs, 20 units)



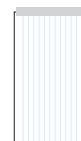
- Paper and writing tool for the student



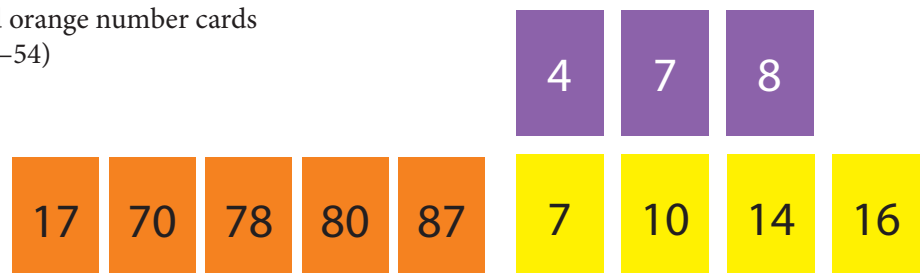
- Two clear plastic bags prepared as follows:
 - Bag A has 6 loose linking cubes (item D13)
 - Bag B has 17 loose linking cubes (item D14)



- Prepare index cards with tape to serve as cover flaps (items D19–20)



- Prepare purple, yellow, and orange number cards (see pp. 5–6, for items D52–54)



- A 9-inch length of string or yarn (items D121–123)



- A supply of about 12 small, uniform-sized paper clips (item D124)



- One large paper clip (item D125)



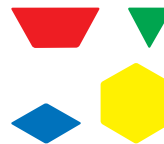
- A ruler (marked in inches; item D126)



- Prepare purple and orange geometric shapes (see pp. 7–8, for items D138–139)



- Pattern blocks (at least 6 triangles, 6 rhombi, 4 trapezoids, 2 hexagons, item D140)



- Additional diagnostic tools:
 - 100 chart
 - number line
 - additional supply of base-ten blocks

4

7

8

7

10

14

16

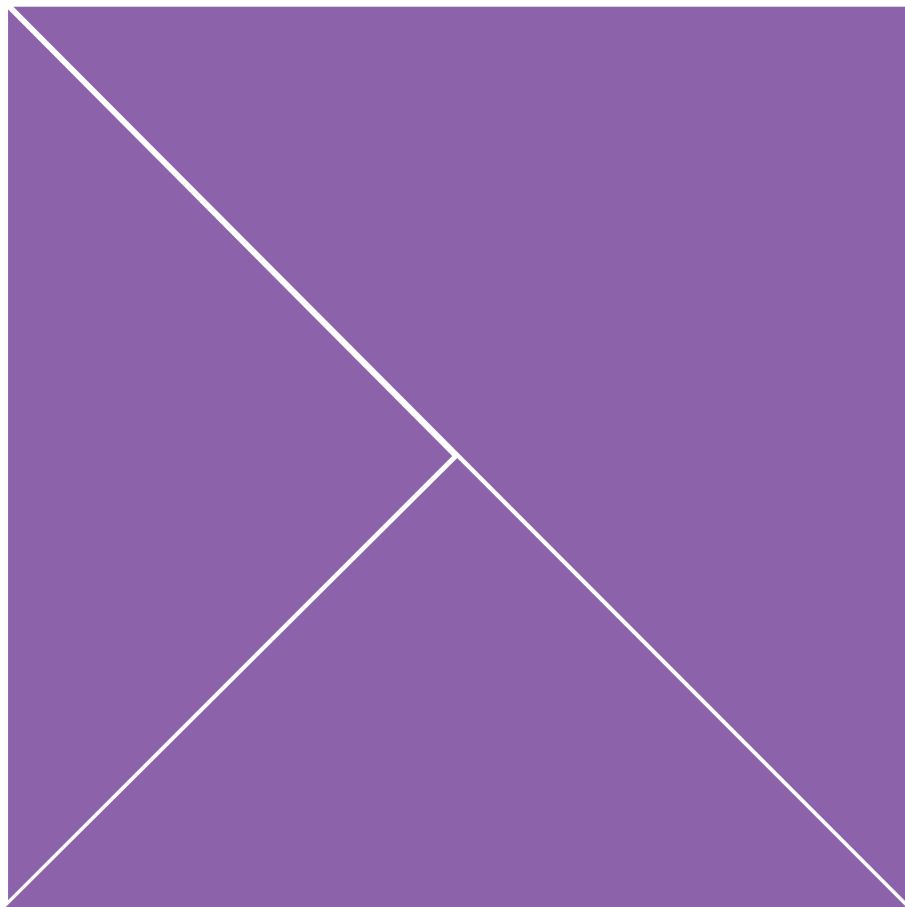
17

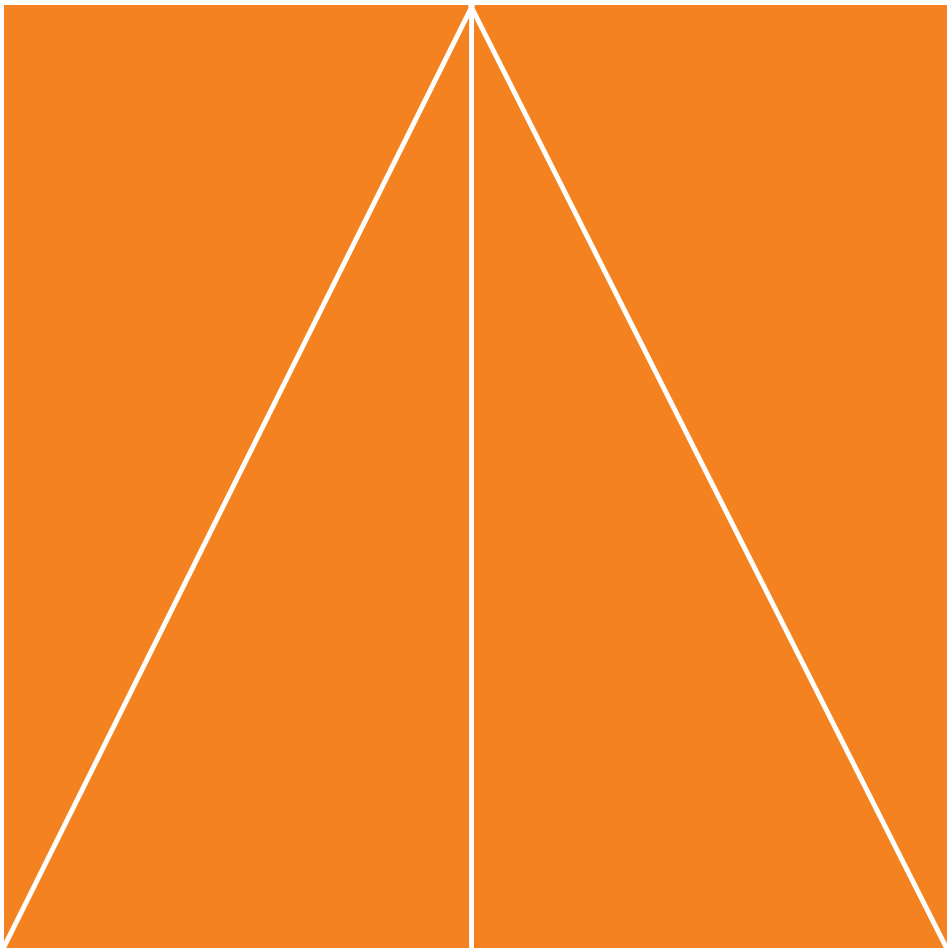
70

78

80

87





Concept Area 1

Competencies for Verbal Counting

Does the student understand and use these big ideas?

- Numbers belong in a sequence that includes various patterns
- Familiarity with the number sequence allows us to count forward or backward from any point in the sequence
- Knowing number patterns allows us to count by numbers other than one

Learning goal	Performance level		
	A	B	C
1.1 Counts by ones	To 10	To 20	To at least 40
1.2 Counts forward from variable starting points	Start value less than 10	Start value in teens	Start value above 50
1.3 Counts backward by ones from variable starting points	<i>From 10</i>	<i>From 20</i>	<i>Start value above 50 (across decades)</i>
1.4 Uses skip counting	<i>By tens to 100</i>	<i>By fives to 55</i>	<i>By twos to 24</i>

Count for Me

Items D1, D2, D3

Verbal Counting

1.1A, 1.1B, 1.1C

Counts by ones (to 10, to 20, to at least 40)

Count for Me

- Say, “Count for me until I say stop.”
- Stop the student when he or she reaches 40.

Materials available

none

Correct response

Accurate count to 10, 20, 40

For further diagnosis

- Listen for skipped numbers.
- Ask the student to start over and count more slowly.
- Does the student use patterns or rhythms in counting?
- Does the student have difficulty distinguishing between teen numbers (e.g., 14 and 15)?

Count From 4 to 11

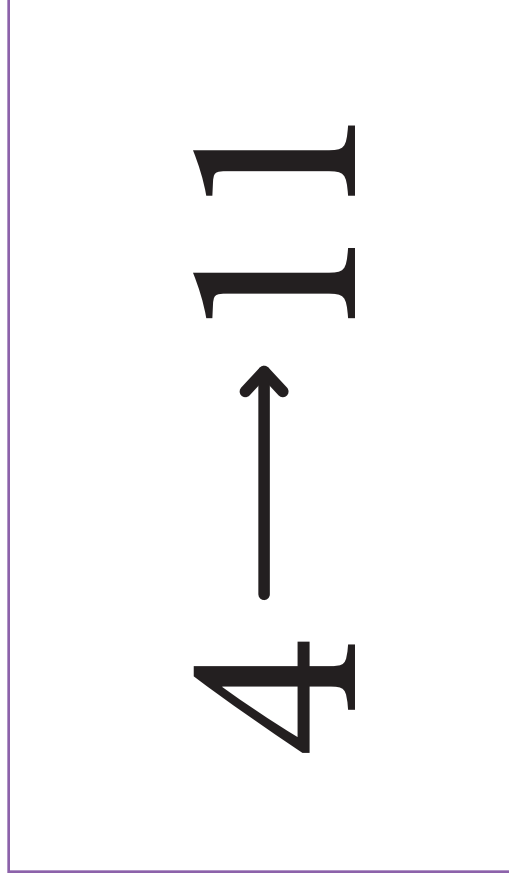
4



11

Count From 4 to 11

- Say, “**Count from 4 to 11.**”
- If necessary, use this prompt: “**Say 4.**”
- Wait for the student to say “4.”
- Say, “**Now keep counting up from 4 until you reach 11.**”



For further diagnosis

- Does the student need a “running start” (counting from one to four and then continuing past four)?
- Does the student count backward when asked to count on from four?
- Ask the student to name the number after various numbers (e.g., “What number comes after six?”).
- Ask the student to count on from a lower or higher number.
- Ask the student to continue past 11.

Materials available

none

Correct response

4, 5, 6, 7, 8, 9, 10, 11

Count From 16 to 22

16 → 22

Count From 16 to 22

- Say, “**Count from 16 to 22.**”
- If necessary, use this prompt: “**Say 16.**”
- Wait for the student to say “16.”
- Say, “**Now keep counting up from 16 until you reach 22.**”

16 → 22

For further diagnosis

- Does the student need a “running start” (counting up to 16 and then continuing past 16)?
- Does the student skip numbers when starting from 16?
- Does the student hesitate at the decade (from 19 to 20)?
- Ask the student to name the number after various numbers (e.g., “What number comes after 12?”).
- Ask the student to count on from a lower number.
- Ask the student to count past 22.

Materials available

none

Correct response

16, 17, 18, 19, 20, 21, 22

Count From 67 to 75

67 → 75

Count From 67 to 75

- Say, “Count from 67 to 75.”
- If necessary, use this prompt: “Say 67.”
- Wait for the student to say “67.”
- Say, “Now keep counting up from 67 until you reach 75.”

67 → 75

Materials available

none

Correct response

67, 68, 69, 70, 71, 72, 73, 74, 75

For further diagnosis

- Does the student skip numbers when starting from 67?
- Does the student hesitate at the decade (from 69 to 70)?
- Ask the student to name the number after various numbers (e.g., “What number comes after 53?”).
- Ask the student to count on from a lower (2-digit) number.
- Ask the student to continue past 75.

Count Backward From 10

10 → 1

Count Backward From 10

- Say, “Count backward from 10 to 1” or “Count down from 10 to 1.”

10 → 1

Materials available

none

Correct response

10, 9, 8, 7, 6, 5, 4, 3, 2, 1

For further diagnosis

- Does the student skip numbers?
- Does the student count slowly as if thinking between each count?
- Ask the student to count down from another number in the sequence (e.g., seven) to see if he or she has a firm command of the reverse number sequence.
- Ask the student to name the number before various numbers (e.g., “What number comes before six?”).

Count Backward From 20

20 → 1

Count Backward From 20

- Say, “Count backward from 20 to 1” or “Count down from 20 to 1.”

20 → 1

Materials available

none

Correct response

20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1

For further diagnosis

- Does the student skip numbers?
- Does the student count slowly as if thinking between each count?
- Ask the student to count down from another number in the sequence (e.g., 14) to see if he or she has a firm command of the reverse number sequence.
- Ask the student to name the number before various numbers (e.g., “What number comes before 17?”).

Count Backward From 92 to 85

92 → 85

Count Backward From 92 to 85

- Say, “Start with 92 and count backwards to 85” or “Count down from 92 to 85.”

92 → 85

For further diagnosis

- Does the student skip numbers?
- Does the student count slowly as if thinking between each count?
- Does the student hesitate crossing the decade (from 90 to 89)?
- Ask the student to count down from other 2-digit numbers lower in the sequence (e.g., 38 or 57).
- Ask the student to name the number that comes before various numbers (e.g., 62).

Materials available

none

Correct response

92, 91, 90, 89, 88, 87, 86, 85

Count by Tens

10

Count by Tens

- Say, “**Count by tens.**”
- If necessary, say, “**It sounds like this: 10, 20, . . .**”
- Stop the student when he or she reaches 100.

10

Materials available

none

For further diagnosis

- Does the student skip numbers?
- Does the student count slowly, counting to 10 between each count?
- Provide a 100 chart and ask the student to find the count by decade numbers.
- Ask the student what he or she notices about the count by 10 numbers (e.g., all end in zero, the tens digit increases by one each time).

Correct response

10, 20, 30, 40, 50, 60, 70, 80, 90, 100

Count by Fives

5

Count by Fives

- Say, “**Count by fives.**”
- If necessary, start the count by saying “**5, 10, . . .**”
- Stop the student when he or she reaches 55.

5

Materials available

none

Correct response

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55

For further diagnosis

- Does the student skip numbers?
- Does the student count slowly, counting to five between each count?
- Provide a 100 chart and ask the student to find the “count by five” numbers.
- Ask the student what he or she notices about the count by five numbers (e.g., all end in five or zero, if there’s a pattern to the count).

Count by Twos

2

Count by Twos

- Say, “**Count by twos.**”
- If necessary, start the count by saying “2, 4,”
- Stop the student when he or she reaches 24.

2

Materials available

none

Correct response

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

For further diagnosis

- Does the student skip numbers?
- Does the student count slowly, counting silently by one between each verbal count?
- Provide a number line and ask the student to find the “count by two” numbers.
- Ask the student what he or she notices about the count by two numbers (e.g., all are even, every other number is represented, skips the odd numbers).

End of Concept Area 1

Concept Area 2

Competencies for Counting Objects

Does the student understand and use these big ideas?

- A collection of objects can be counted to find out how many are in it
- Counting by numbers other than one allows us to count faster and more reliably
- Some collections are easily recognizable and we know how many are included in them without counting

Learning goal	Performance level		
	A	B	C
2.1 Counts objects in a given collection	To 10	To 20	To 100 with objects grouped in tens
2.2 Produces a collection of a specified size	To 10	To 20	To 100
2.3 Recognizes collections arranged in patterns without counting	To 4	To 6	To 10
2.4 Writes the numeral to represent a quantity	To 10	To 20	To 100

How Many Cubes in Bag A?

Item D13

Counting Objects

2.1A

Counts objects in a given collection (to 10)

32

How Many Cubes in Bag A?

- Use Bag A with six cubes.
- Hand Bag A to the student.
- Say, **“How many cubes are in this bag? You may take the cubes out of the bag, if you would like.”**

For further diagnosis

- Help the student organize the cubes for the count (e.g., suggest that he or she move each one from the bag to the page as he counts).
- Remove one cube from the bag and ask the student to count. Do this again until you reach a number the student can accurately count.
- Does the student double tag?
- Does the student recognize small quantities (two, three, or four) without counting?
- Does the student understand that the last number in the count tells how many are in the set?



Materials available



Correct response

6 (cubes)

How Many Cubes in Bag B?

How Many Cubes in Bag B?

- Use Bag B with 17 cubes.
- Hand Bag B to the student.
- Say, “**How many cubes are in this bag? You may take the cubes out of the bag, if you would like.**”

For further diagnosis

- Help the student organize the cubes for the count (e.g., suggest that he or she move each one from the bag to the page as he counts).
- Does the student double tag?
- Does the student recognize small quantities (two, three, or four) without counting?
- Does the student understand that the last number in the count tells how many are in the set?
- Does the student count by twos or other numbers?

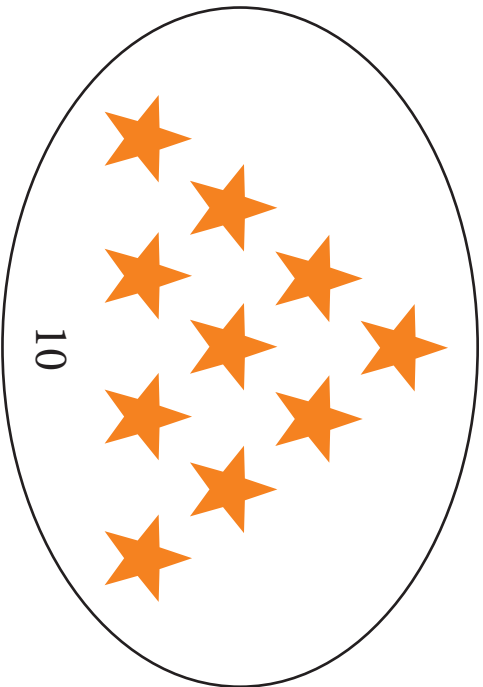
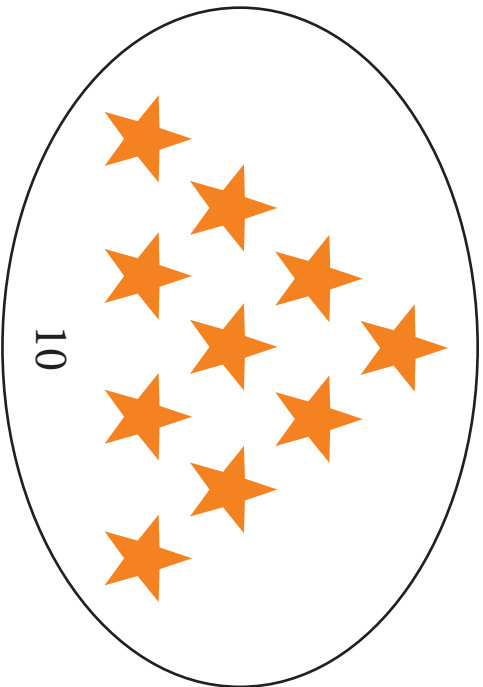
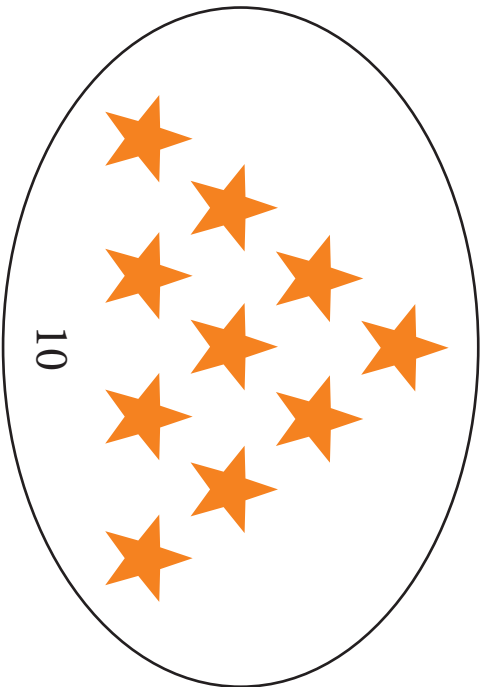
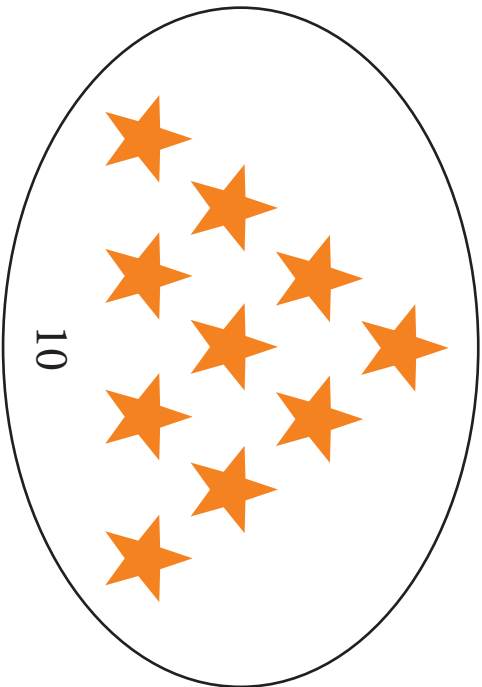
Materials available



Correct response

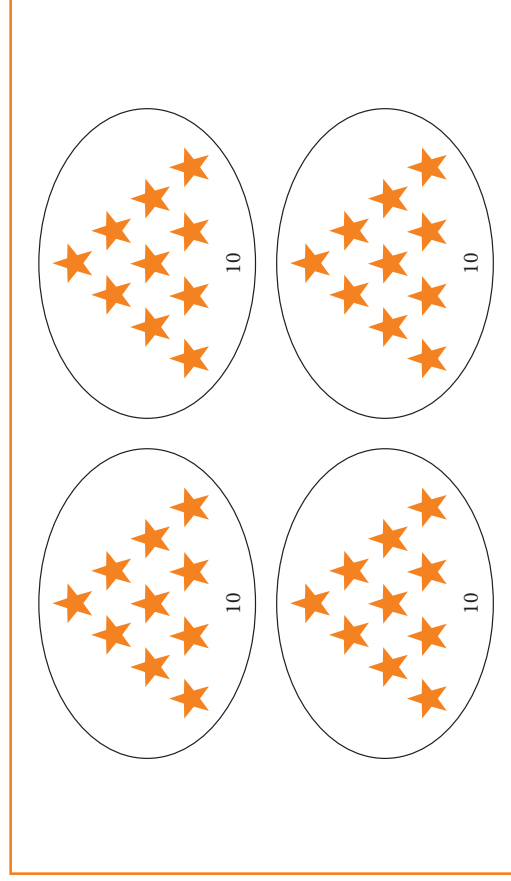
17 (cubes)

How Many Stars?



How Many Stars?

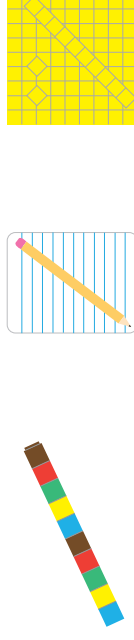
- Say, “How many stars are on this page?”
- If the student counts all by ones say, “Can you count a faster way?”



For further diagnosis

- Does the student count by ones?
- Does the student have to start over more than once?
- Does the student start out counting by ones and then switch to counting by tens after recognizing the pattern?
- Suggest counting by tens if the student does not use this as the “faster way.”
- Ask the student to use base-ten blocks to represent the stars in each group and then count the total.

Materials available



Correct response

Counts by 10 and says “40”

Place Nine Cubes on the Page



Place Nine Cubes on the Page

- Place the supply of loose cubes within easy reach.
- Say, “**Place 9 cubes on the page.**”
- When the student is done, slide the cubes off the page keeping them in a group.

For further diagnosis

- Help the student organize the cubes for the count (e.g., suggest that he or she move each one onto the page as he or she counts).
- Does the student double tag?
- Does the student recognize small quantities (two, three, or four) without counting?
- Does the student understand that the last number in the count tells how many are in the set?
- Does the student recount the total with each new cube added to the collection?

9

Materials available



Correct response

Places 9 cubes

Place 16 Cubes on the Page

16

Place 16 Cubes on the Page

- Place the supply of loose cubes and the nine cubes from the previous problem within easy reach.
- Say, “Place 16 cubes on the page.”

16

For further diagnosis

- Does the student start with the nine cubes from the previous task and count on from nine?
- Does the student double tag?
- Does the student recognize small quantities (two, three, or four) without counting?
- Does the student understand that the last number in the count tells how many are in the set?
- Does the student recount the total with each new cube added to the collection?

Materials available



Correct response

Places 16 cubes

Place 36 Cubes on the Page

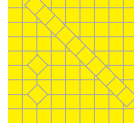
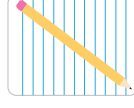
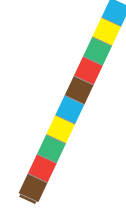
36

Place 36 Cubes on the Page

- Say, “Make a collection of 36 cubes. Use the sticks of 10 and loose cubes.”

36

Materials available



For further diagnosis

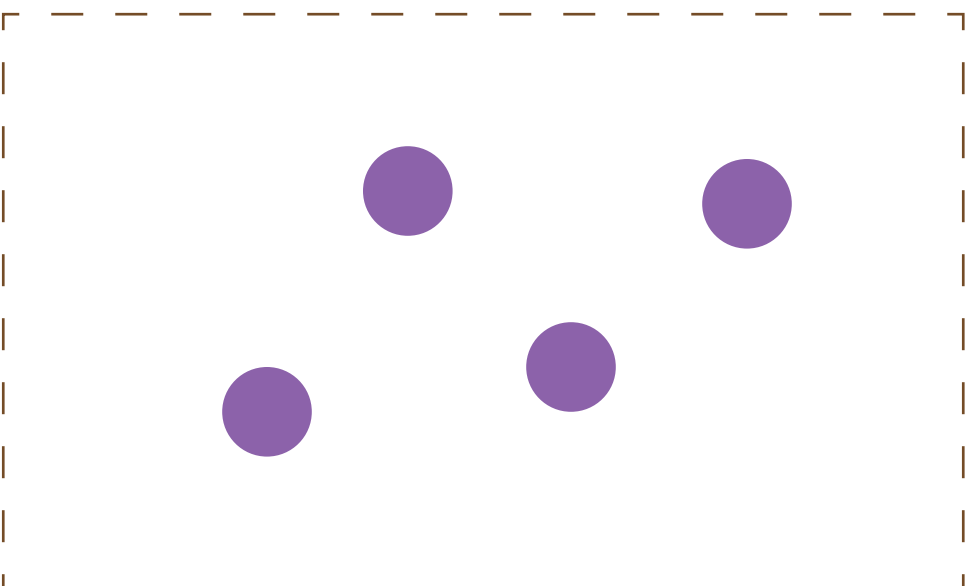
- Does the student count by ones?
- Have the student count a ten-stick to verify that it has 10 cubes.
- Ask the student to verbally count by tens.
- Ask the student how counting by tens could make this task go faster.

Correct response

Places 3 ten-sticks and 6 units on the page

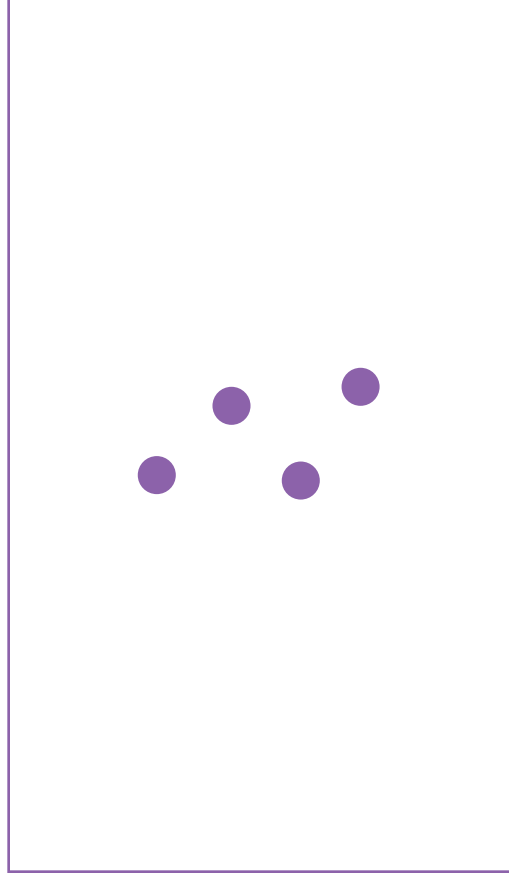
How Many Dots?

Tape index card here



How Many Dots?

- Say, “I will let you quickly peek at some dots. You can look, but you can’t touch them. Tell me how many dots you see.”
- Uncover the dot pattern for a count of three, then recover it.
- Say, “How many dots did you see?”



Materials available

none

Correct response

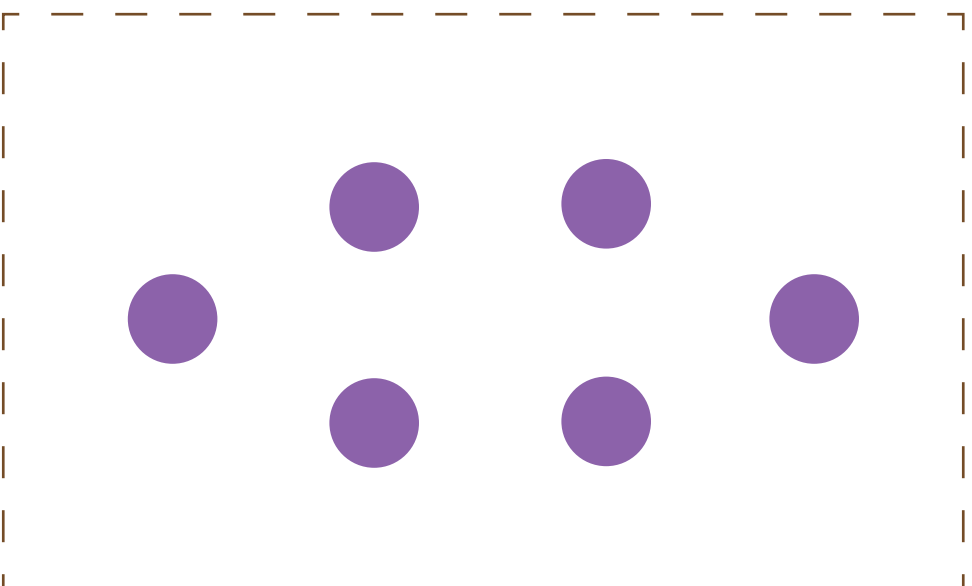
4 (dots)

For further diagnosis

- Ask the student what he or she saw.
- Does the student see two rows (vertical or diagonal) of two?
- Does the student see a triangle of three and one more?
- Does the student “just know” without seeing a particular pattern?
- Ask the student to show values up to five on fingers in multiple ways (e.g., “Show me four another way.”).

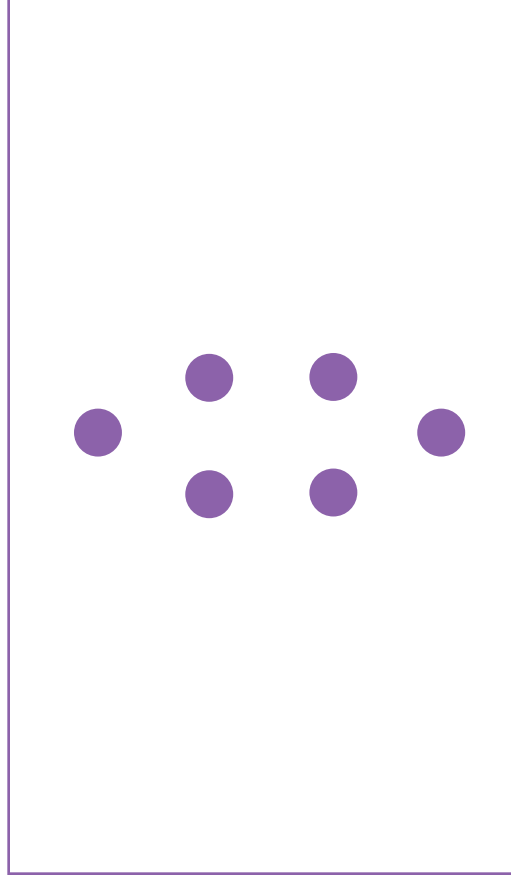
How Many Dots?

Tape index card here



How Many Dots?

- Say, “I will let you quickly peek at some dots. You can’t touch them. How many do you see?”
- Uncover the dot pattern for a count of three, then recover it.
- Say, “How many dots did you see?”



For further diagnosis

- Ask the student what he or she saw.
- Does the student see two triangles (top and bottom) of three dots each?
- Does the student see a square in the middle with an extra dot at the top and bottom?
- Does the student “just know” without seeing a particular pattern?
- Ask the student to show values up to six on fingers in multiple ways (e.g., “Show me six another way.”).

Materials available

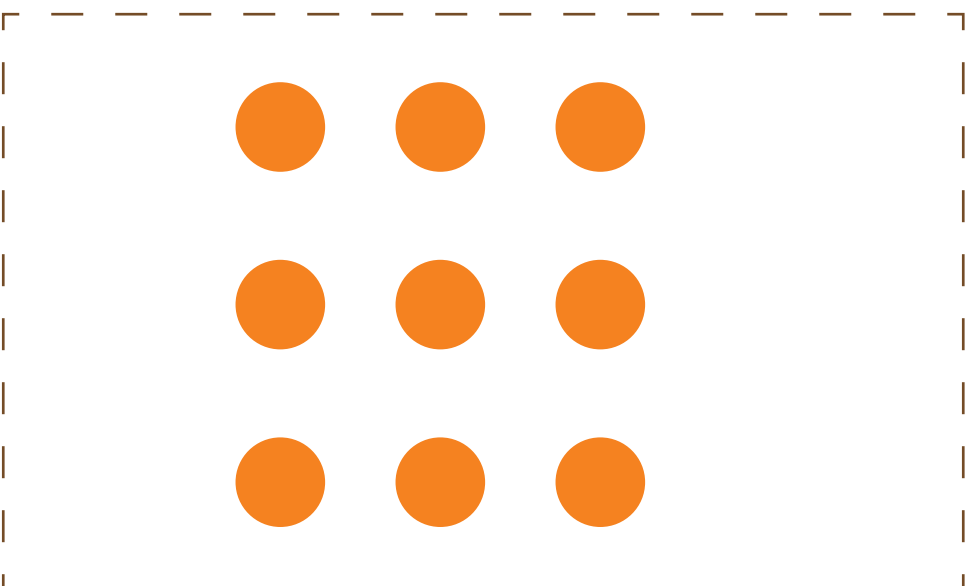
none

Correct response

6 (dots)

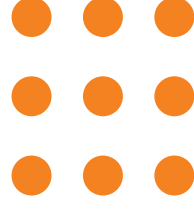
How Many Dots?

Tape index card here



How Many Dots?

- Say, “I will let you quickly peek at some dots. You can look, but you can’t touch them. Tell me how many dots you see.”
- Uncover the dot pattern for a count of three, then recover it.
- Say, “How many dots did you see?”



Materials available

none

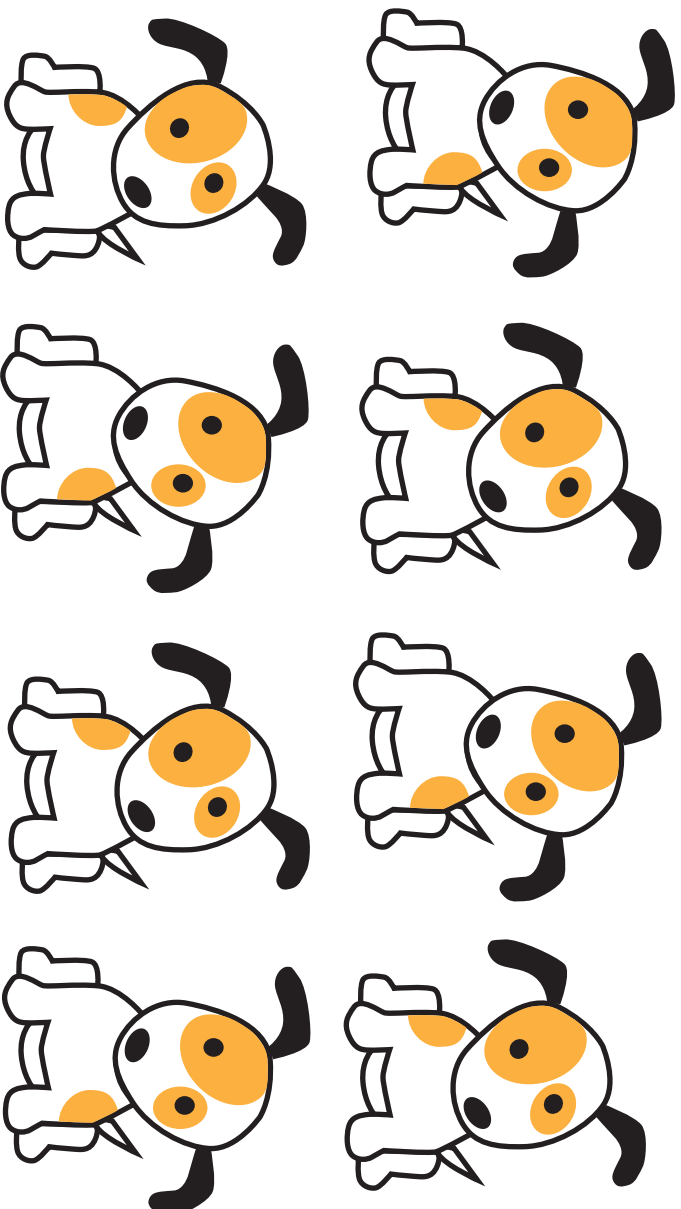
Correct response

9 (dots)

For further diagnosis

- Ask the student what he or she saw.
- Does the student see three rows or columns of three?
- Does the student say “six” (thinking three and three)?
- Does the student see a diagonal of three and two triangles of three?
- Does the student “just know” without seeing a particular pattern?

Write the Number of Dogs

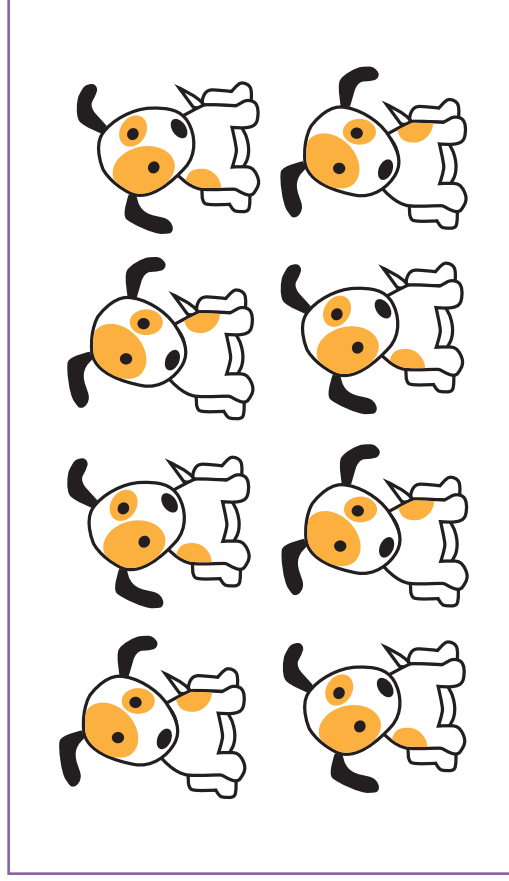


Write the Number of Dogs

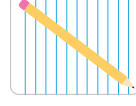
- Say, “Count the dogs on this page. Write the number on this paper.”
- Allow the student to touch the pictures on the page.
- If the student counts incorrectly, but correctly writes the number he or she says, score *p* (*partially correct*).

For further diagnosis

- Does the student count correctly but write the number incorrectly?
- Does the student count by ones? Twos? Fours?
- Does the student miscount, but write the correct number for his or her count?
- Ask the student to put a cube on each dog, then count the cubes.



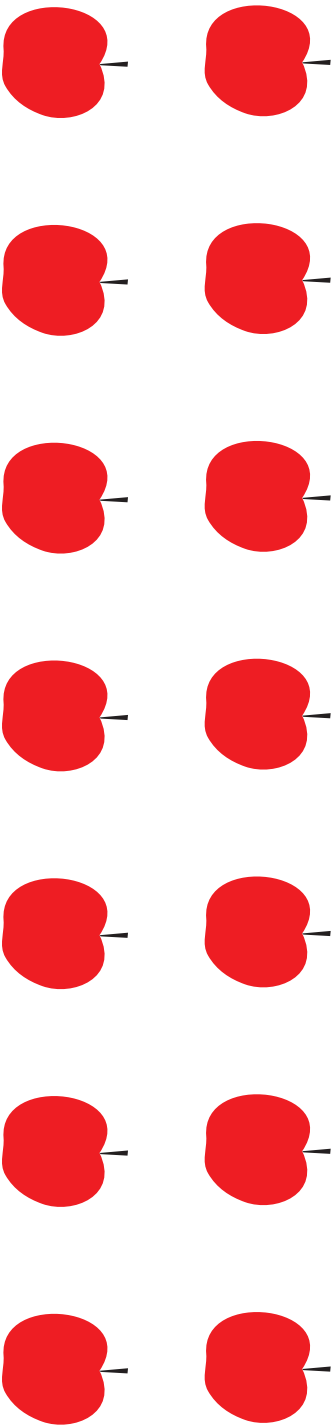
Materials available



Correct response

Writes “8”

Write the Number of Apples

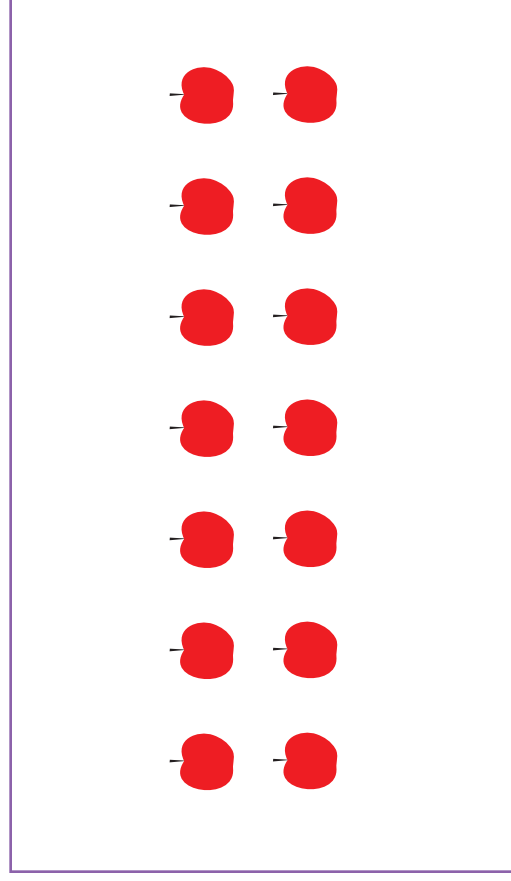


Write the Number of Apples

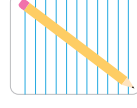
- Say, “Count the apples. Write the number on this paper.”
- Allow the student to touch the pictures on the page.
- If the student counts incorrectly, but correctly writes the number he or she says, score *p* (*partially correct*).

For further diagnosis

- Does the student count correctly but write the number incorrectly?
- Does the student count by ones? Twos? Another number?
- Does the student miscount, but write the correct number for his or her count?
- Ask the student to place a cube on each apple then count the cubes.



Materials available



Correct response

Writes “14”

Write the Number of Pencils

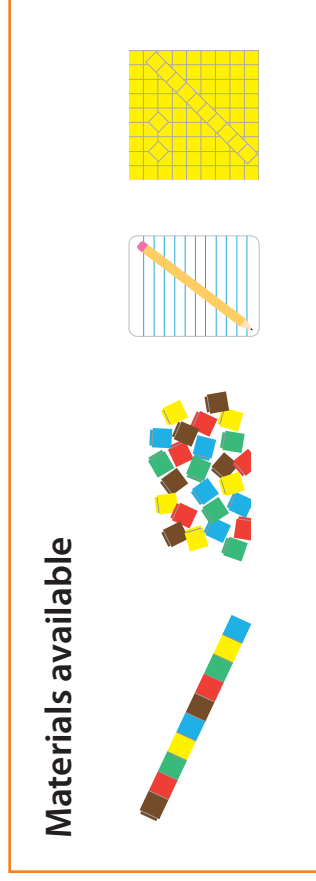
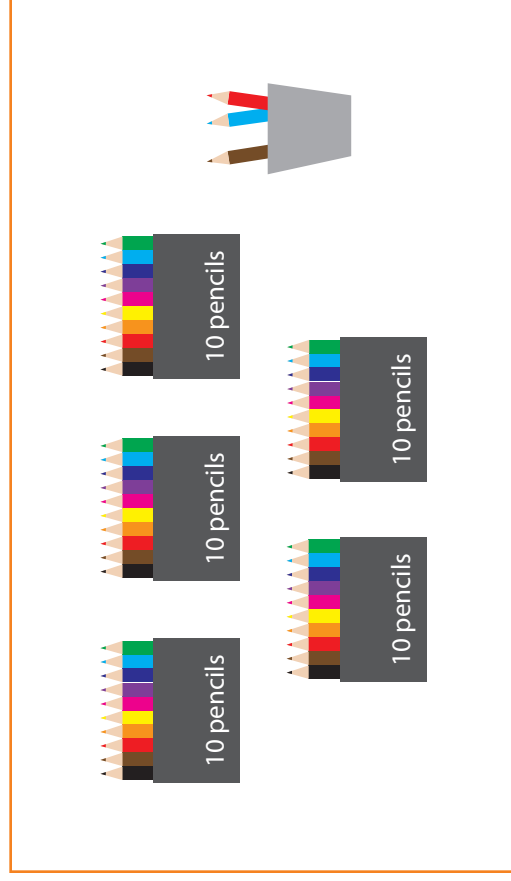


Write the Number of Pencils

- Say, “This picture shows some pencils in boxes and some pencils in a cup (point to the cup). How many pencils are in this picture? Write the number on this paper.”
- If the student counts by ones say, “Can you count a faster way?”

For further diagnosis

- Suggest counting by tens if the student counts by ones.
- Can the student accurately switch from counting by tens to counting by ones?
- Does the student count correctly but write the number incorrectly?
- Does the student miscount, but write the correct number for his or her count?
- Write “53.” Ask the student how the boxes and loose pencils correspond to the digits.



Correct response

Uses tens and ones and writes “53”

End of Concept Area 2

Concept Area 3

Competencies for Adding to and Taking From in Contexts

Does the student understand and use these big ideas?

- Physical objects or drawings can be used to solve problems involving joining, separating, comparing, and considering parts of a whole collection
- Counting forward or backward can be used to solve problems involving joining, separating, comparing, and considering parts of a whole collection
- Number relationships and known number facts can be used to solve problems involving joining, separating, comparing, and considering parts of a whole collection

Learning goal	Performance level		
	A	B	C
3.1 Solves context problems of the type join, result unknown (JRU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.2 Solves context problems of the type separate, result unknown (SRU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.3 Solves context problems of the type part-part-whole, whole unknown (PPW-WU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.4 Solves context problems of the type part-part-whole, part unknown (PPW-PU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.5 Solves context problems of the type separate, change unknown (SCU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.6 Solves context problems of the type compare, difference unknown (CDU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.7 Solves context problems of the type separate, start unknown (SSU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.8 Solves context problems of the type join, start unknown (JSU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18

How Many Crayons?

I had 3 crayons.

My friend gave me 2 more crayons.

Now how many crayons do I have?

How Many Crayons?

- Read the problem aloud.

I had 3 crayons.
My friend gave me 2 more crayons.
Now how many crayons do I have?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

5 (crayons)

How Many Apples?

Mom had 3 apples.

She bought 6 more apples.

Now how many apples does Mom have?

How Many Apples?

- Read the problem aloud.

Mom had 3 apples.
She bought 6 more apples.
Now how many apples does Mom have?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? From first? From larger?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

9 (apples)

How Many Carrots?

Dad had 9 carrots.

He bought 4 more carrots.

Now how many carrots does Dad have?

How Many Carrots?

- Read the problem aloud.

Dad had 9 carrots.
He bought 4 more carrots.
Now how many carrots does Dad have?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use 10 as a referent?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

13 (carrots)

How Many Birds?

I saw 6 birds on the roof.

Two flew away.

How many birds are still on the roof?

How Many Birds?

- Read the problem aloud.

I saw 6 birds on the roof.
Two flew away.
How many birds are still on the roof?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

4 (birds)

How Many Cars?

There were 8 cars in the parking lot.

Five of them drove away.

How many cars are in the parking lot now?

How Many Cars?

- Read the problem aloud.

There were 8 cars in the parking lot.
Five of them drove away.
How many cars are in the parking lot now?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

3 (cars)

How Many Cookies?

There were 11 cookies on the plate.

We took 6 of them.

How many cookies are on the plate now?

How Many Cookies?

- Read the problem aloud.

There were 11 cookies on the plate.
We took 6 of them.
How many cookies are on the plate now?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use 10 as a referent?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

5 (cookies)

How Many Reptiles?

There are 2 lizards and 2 turtles at the pet store. How many reptiles are at the pet store?

How Many Reptiles?

- Read the problem aloud.
- If necessary, explain that both lizards and turtles are reptiles.

There are 2 lizards and 2 turtles at the pet store. How many reptiles are at the pet store?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

4 (reptiles)

How Many People?

There are 3 adults and 6 children at the park.
How many people are at the park?

How Many People?

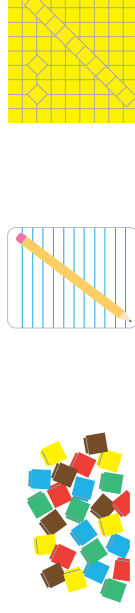
- Read the problem aloud.

There are 3 adults and 6 children at the park. How many people are at the park?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? From first? From larger?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

9 (people)

How Many Coins?

I have 8 pennies in one pocket and 9 pennies in the other pocket. How many pennies do I have in my pockets?

How Many Coins?

- Read the problem aloud.

I have 8 pennies in one pocket and 9 pennies in the other pocket. How many pennies do I have in my pockets?

For further diagnosis

- If it is not obvious, ask how the student figured out the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use 10 as a referent?
- Ask the same problem different numbers (larger or smaller values).

Materials available



Correct response

17 (pennies)

How Many Green Apples?

We bought 6 apples. Two of them are red apples and the rest are green apples. How many green apples did we buy?

How Many Green Apples?

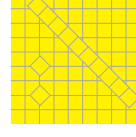
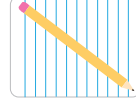
- Read the problem aloud.

We bought 6 apples. Two of them are red apples and the rest are green apples. How many green apples did we buy?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

4 (green apples)

How Many Yellow Shirts?

My sister has 8 shirts. Six of them are blue and the rest are yellow. How many yellow shirts does my sister have?

How Many Yellow Shirts?

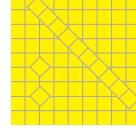
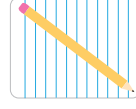
- Read the problem aloud.

My sister has 8 shirts. Six of them are blue and the rest are yellow. How many yellow shirts does my sister have?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

2 (yellow shirts)

How Many Girls?

There are 13 players on my soccer team.
Five of them are boys and the rest of them
are girls. How many girls are on my soccer
team?

How Many Girls?

- Read the problem aloud.

There are 13 players on my soccer team.
Five of them are boys and the rest of
them are girls. How many girls are on
my soccer team?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use 10 as a referent?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

8 (girls)

How Many Toy Trucks?

My brother had 4 toy trucks. He lost some at the park. Now he has 1 toy truck. How many toy trucks did he lose at the park?

How Many Toy Trucks?

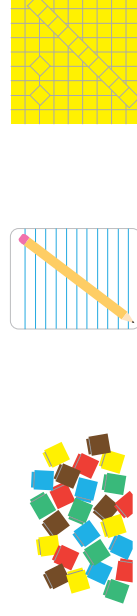
- Read the problem aloud.

My brother had 4 toy trucks. He lost some at the park. Now he has 1 toy truck. How many toy trucks did he lose at the park?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

3 (toy trucks)

How Many Stickers?

I had 8 stickers. I gave some to my friend.
Now I have 3 stickers. How many stickers
did I give to my friend?

How Many Stickers?

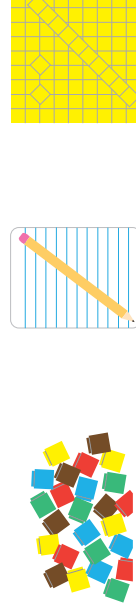
- Read the problem aloud.

I had 8 stickers. I gave some to my friend. Now I have 3 stickers. How many stickers did I give to my friend?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

5 (stickers)

How Many Pencils?

There were 11 pencils in my backpack. On my way to school I lost some of them. Now I only have 7 pencils in my backpack. How many pencils did I lose?

How Many Pencils?

- Read the problem aloud.

There were 11 pencils in my backpack.
On my way to school I lost some of them.
Now I only have 7 pencils in my backpack.
How many pencils did I lose?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use 10 as a referent?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

4 (pencils)

How Many More Boys?

In my family there are 5 boys and 3 girls.
How many more boys are there?

How Many More Boys?

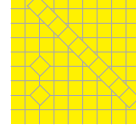
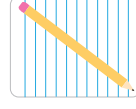
- Read the problem aloud.

In my family there are 5 boys and 3 girls.
How many more boys are there?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using smaller numbers.

Materials available



Correct response

2 (more boys)

How Many More Goats?

There are 10 horses and 4 goats on the farm.
How many more goats should the farmer
buy so that there will be the same number
of goats and horses?

How Many More Goats?

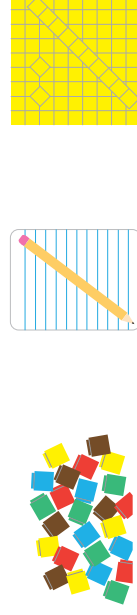
- Read the problem aloud.

There are 10 horses and 4 goats on the farm. How many more goats should the farmer buy so that there will be the same number of goats and horses?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

6 (more goats)

How Many More Peach Trees?

A farmer planted 16 apple trees and 9 peach trees. How many more peach trees should the farmer plant so there will be the same number of peach trees and apple trees?

How Many More Peach Trees?

- Read the problem aloud.

A farmer planted 16 apple trees and 9 peach trees. How many more peach trees should the farmer plant so there will be the same number of peach trees and apple trees?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on? Count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use 10 as a referent?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

7 (more peach trees)

How Many Brownies?

There were some brownies on a plate. I ate two of them. Now there are 3 brownies on the plate. How many brownies were on the plate to start?

How Many Brownies?

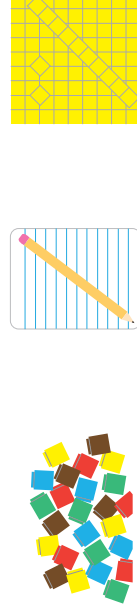
- Read the problem aloud.

There were some brownies on a plate. I ate two of them. Now there are 3 brownies on the plate. How many brownies were on the plate to start?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use trial and error?
- Ask the same problem using different numbers (larger or smaller numbers).

Materials available



Correct response

5 (brownies)

How Many Paintbrushes?

There were some paintbrushes in the art center. My teacher took 7 of them for an art project. Now there are 2 paintbrushes in the art center. How many paintbrushes were in the art center to start?

How Many Paintbrushes?

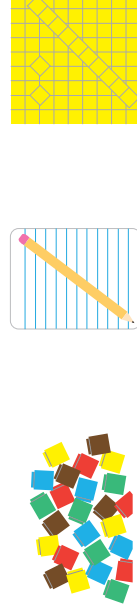
- Read the problem aloud.

There were some paintbrushes in the art center. My teacher took 7 of them for an art project. Now there are 2 paintbrushes in the art center. How many paintbrushes were in the art center to start?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use trial and error?
- Ask the same problem using smaller numbers.

Materials available



Correct response

9 (paintbrushes)

How Many Birds?

There were some birds in a tree. Eight of them flew away. Now there are 3 birds in the tree. How many birds were in the tree to start?

How Many Birds?

- Read the problem aloud.

There were some birds in a tree. Eight of them flew away. Now there are 3 birds in the tree. How many birds were in the tree to start?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count on?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use trial and error?
- Ask the same problem using different numbers (larger and smaller values).

Materials available



Correct response

11 (birds)

How Many Pencils?

I had some pencils on my desk.

My teacher gave me 1 more pencil and now I
have 5 pencils.

How many pencils did I have to start?

How Many Pencils?

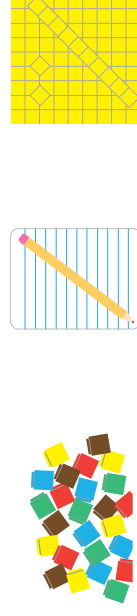
- Read the problem aloud.

I had some pencils on my desk.
My teacher gave me 1 more pencil and
now I have 5 pencils.
How many pencils did I have to start?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use trial and error?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

4 (pencils)

How Many Trees?

We had some trees in our yard. My family planted 5 new trees. Now we have 8 trees in our yard. How many trees did we have in our yard to start?

How Many Trees?

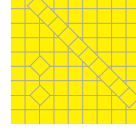
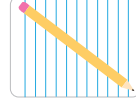
- Read the problem aloud.

We had some trees in our yard. My family planted 5 new trees. Now we have 8 trees in our yard. How many trees did we have in our yard to start?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use trial and error?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

3 (trees)

How Many Candies?

I had some candies in a bag.

My friend gave me 6 more candies and now
I have 14 candies.

How many candies did I have to start?

How Many Candies?

- Read the problem aloud.

I had some candies in a bag.
My friend gave me 6 more candies and
now I have 14 candies.
How many candies did I have to start?

For further diagnosis

- If it is not obvious, ask how the student figured the answer.
- Does the student solve by direct modeling (e.g., using fingers, manipulatives, or pictures)?
- Does the student count back?
- Does the student know the result without calculating (known fact or derived fact)?
- Does the student use trial and error?
- Ask the same problem using different numbers (larger or smaller values).

Materials available



Correct response

8 (candies)

End of Concept Area 3

Concept Area 4

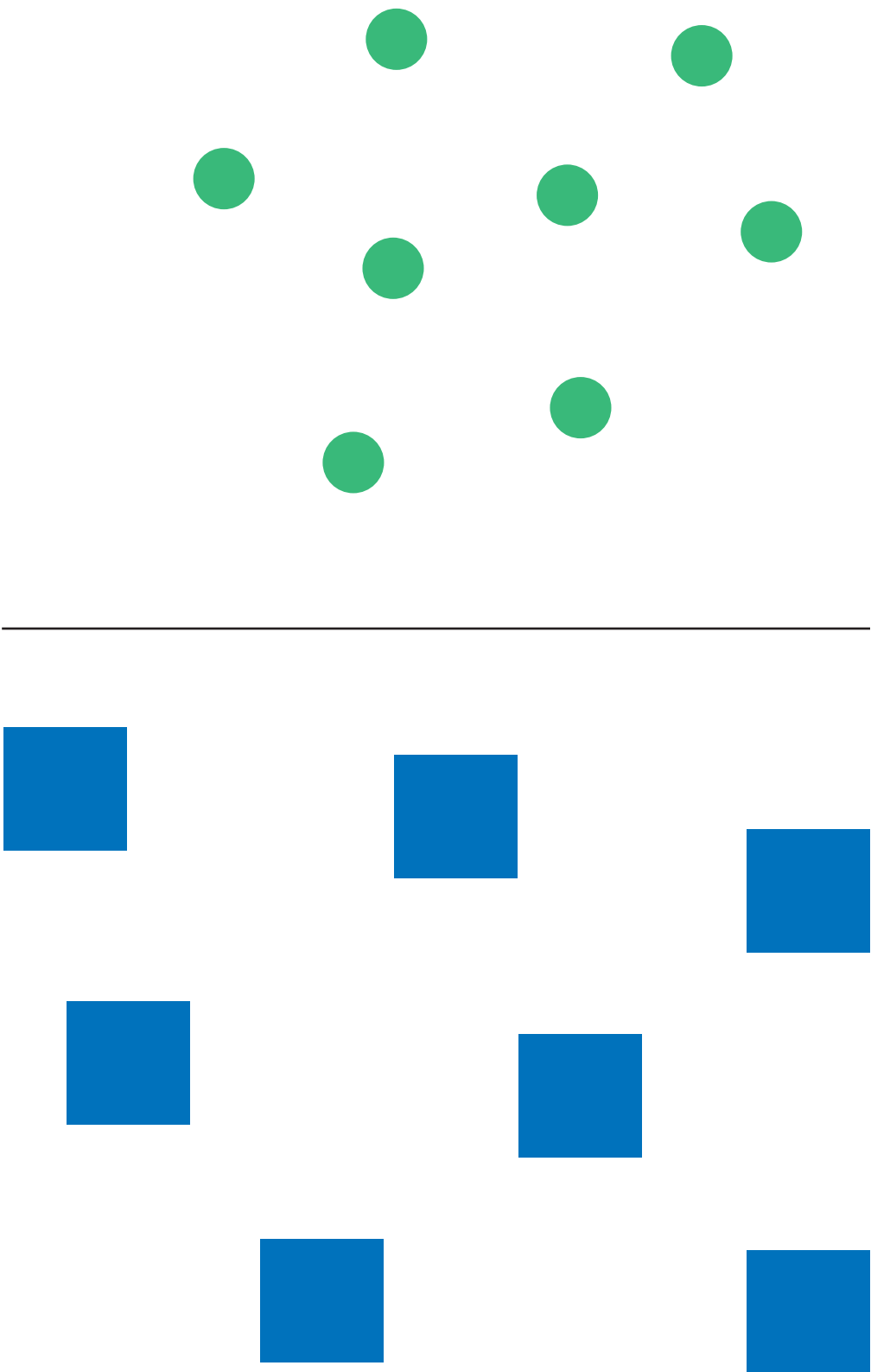
Competencies for Comparing and Ordering Numbers

Does the student understand and use these big ideas?

- The relationship among numbers can be described and represented in multiple ways
- Any group of numbers can be placed in order by magnitude even if they are not adjacent in the counting sequence
- Ordinal numbers tell the position of an element in a sequence

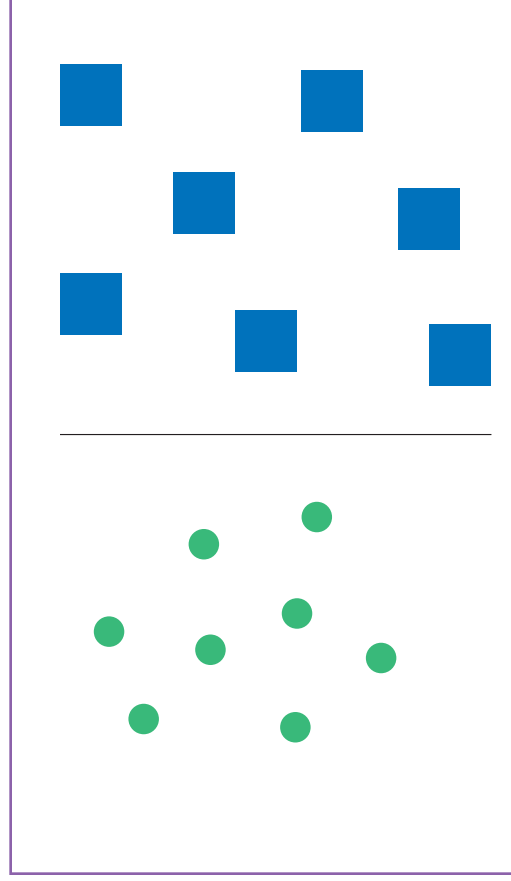
Learning goal	Performance level		
	A	B	C
4.1 Compares sets or numbers	Sets to 10	Sets vs. numbers to 20	Numbers to 100
4.2 Orders 3 or more numbers	3 numbers less than 10	4 numbers to 20	5 numbers to 100
4.3 Represents numbers on the number line	<i>To 10, with grid marks</i>	<i>To 20, with grid marks</i>	<i>To 100, with only decade grid marks</i>
4.4 Identifies ordinal position	<i>To 10th</i>	<i>To 30th</i>	<i>Reads ordinal terms (words) through 9th and uses them</i>
4.5 Determines how many more or less	<i>By comparing sets (to 10)</i>	<i>By counting on (numbers to 20, differences within 5)</i>	<i>By adding or subtracting, to 100</i>

Which Side Has More?



Which Side Has More?

- Say, “Which side has more shapes? The side with green circles or the side with blue squares?”



Materials available



Correct response

Green (left side)

For further diagnosis

- Ask the student how he or she knows the answer.
- Does the student choose blue because the squares are larger and more spread out?
- Ask the student to count to verify.
- Have the student place a linking cube on each square and circle, then make trains and compare their lengths.
- Ask a similar question using fewer items.

Which Has More?



Which Has More?

- Say, “There are pennies outside the box and there are pennies inside the box. Are there more pennies outside or inside the box?”



For further diagnosis

- Ask the student how he or she knows the answer.
- Does the student choose the pennies outside the box because they take up more area?
- Does the student count the pennies accurately?
- Does the student count the pennies by twos, threes, or fours?
- Ask how many would need to be added to the outside to make the sets equal.

Materials available



Correct response

More pennies inside the box

Which Is Largest?

Point to the largest number

44, 66, 64, 46

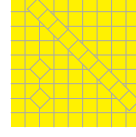
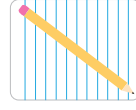
Which Is Largest?

- Say, “Point to the largest number.”

Point to the largest number

44, 66, 64, 46

Materials available



For further diagnosis

- Ask the student how he or she knows the answer.
- Does the student consider both the tens and ones digits in making his or her determination?
- Does the student use the counting sequence as his or her referent? (66 comes later in the counting sequence)

Correct response

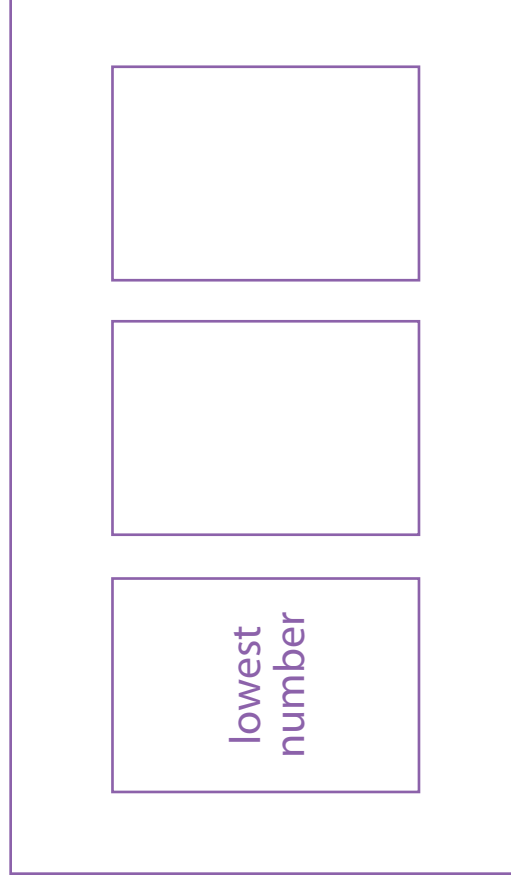
66

Place the Cards in Order

lowest number		
------------------	--	--

Place the Cards in Order

- Hand the student the three purple cards.
- Say, **“Place these cards in number order. Put the lowest (smallest) number here (point to the spot), the middle number here (point), and the highest (largest) number here (point).”**



Materials available

prepared number cards

8 4 7

For further diagnosis

- Ask the student how he or she knows the answer.
- Does the student say the counting sequence in order to determine the order of the numbers?
- Hand the student a number line or 100 chart. Ask him or her to find these numbers on it.
- Ask the student to name the missing cards in the sequence.

Correct response

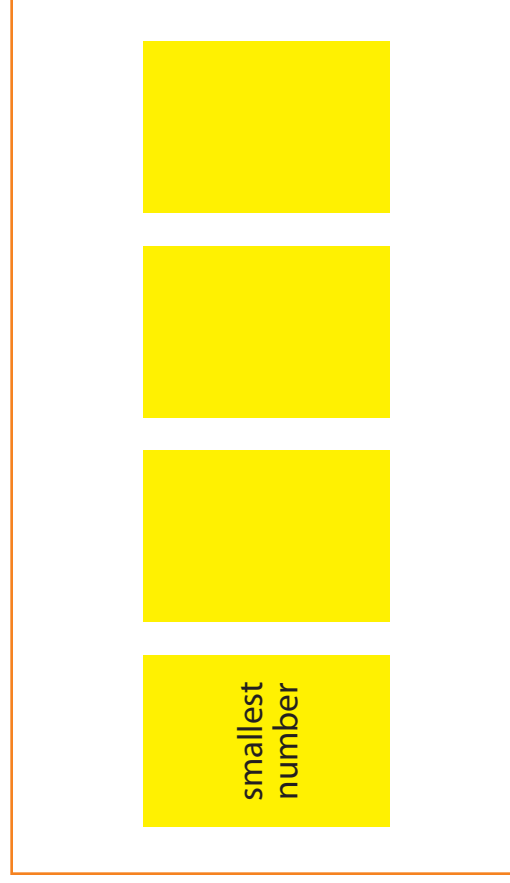
4, 7, 8

Place the Cards in Order

smallest
number

Place the Cards in Order

- Hand the student the yellow number cards.
- Say, “Place these cards in number order.”



For further diagnosis

- Ask the student how he or she knows the answer.
- Does the student say the counting sequence in order to determine the order of the numbers?
- Does the student misplace seven at the far right because it is the largest digit of zero, one, four, six, and seven?
- Hand the student a number line or 100 chart. Ask him or her to find these numbers on it.
- Ask the student what numbers could go between any two of the cards.

Materials available



prepared number cards

Correct response

7, 10, 14, 16

Place the Cards in Order

smallest
number

Place the Cards in Order

- Hand the student the orange number cards.
- Say, “Place these cards in number order.”

smallest number					
--------------------	--	--	--	--	--

Materials available

prepared number cards

70	87	78	17	80
----	----	----	----	----

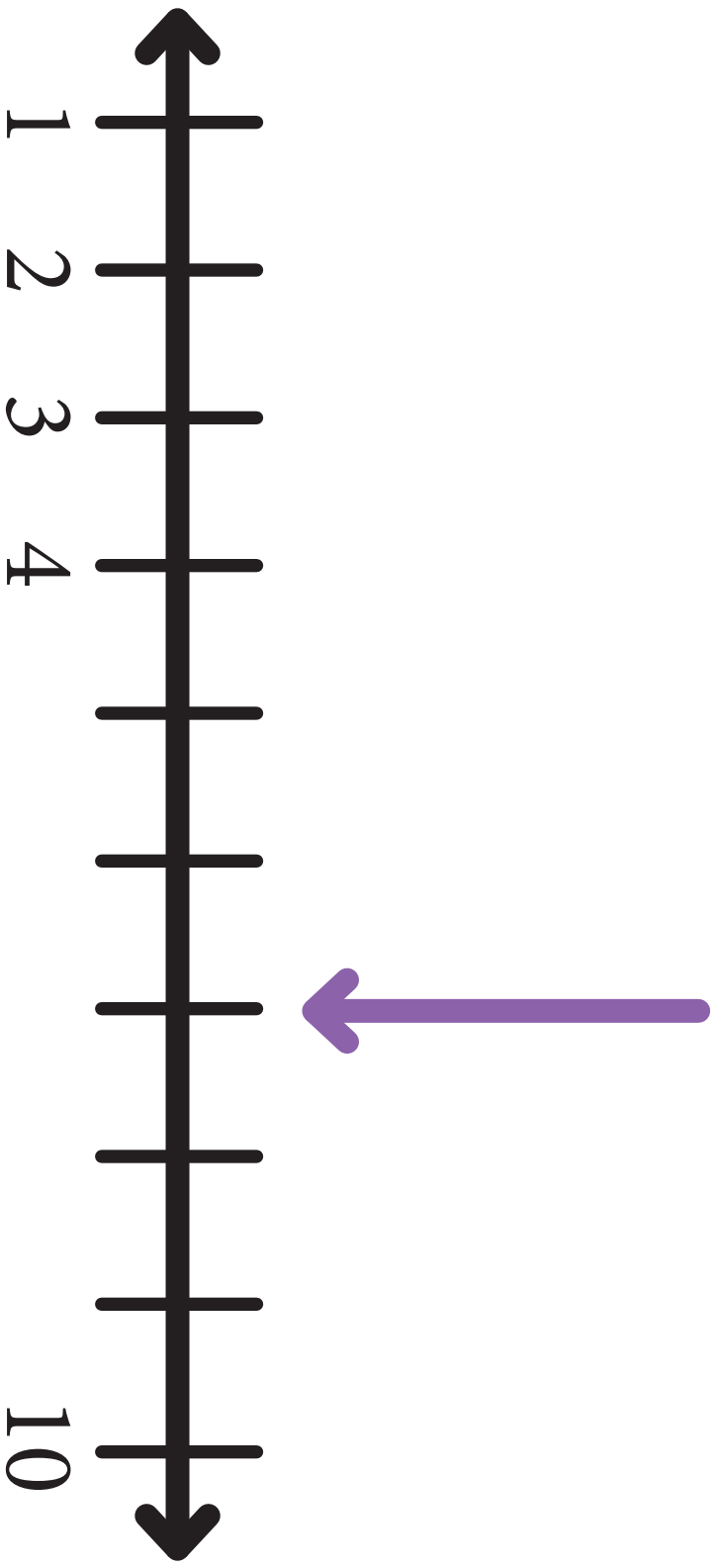
For further diagnosis

- Ask the student how he or she knows the answer.
- Do the place value reversals confuse the student?
- Do the decade numbers confuse the student?
- Hand the student a number line. Ask him or her to find these numbers on it.

Correct response

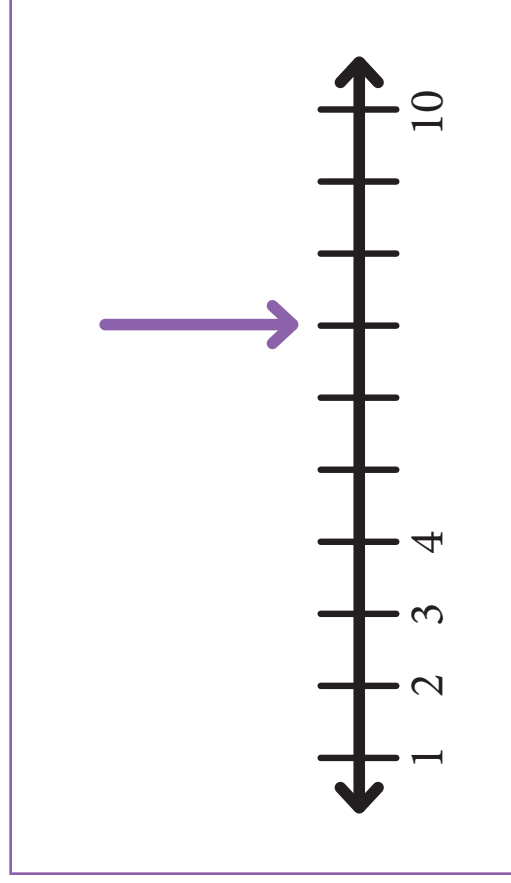
17, 70, 78, 80, 87

What Number is Below the Arrow?



What Number is Below the Arrow?

- Say, “This number line is missing some numbers. What number should be written below the arrow?”



Materials available

none

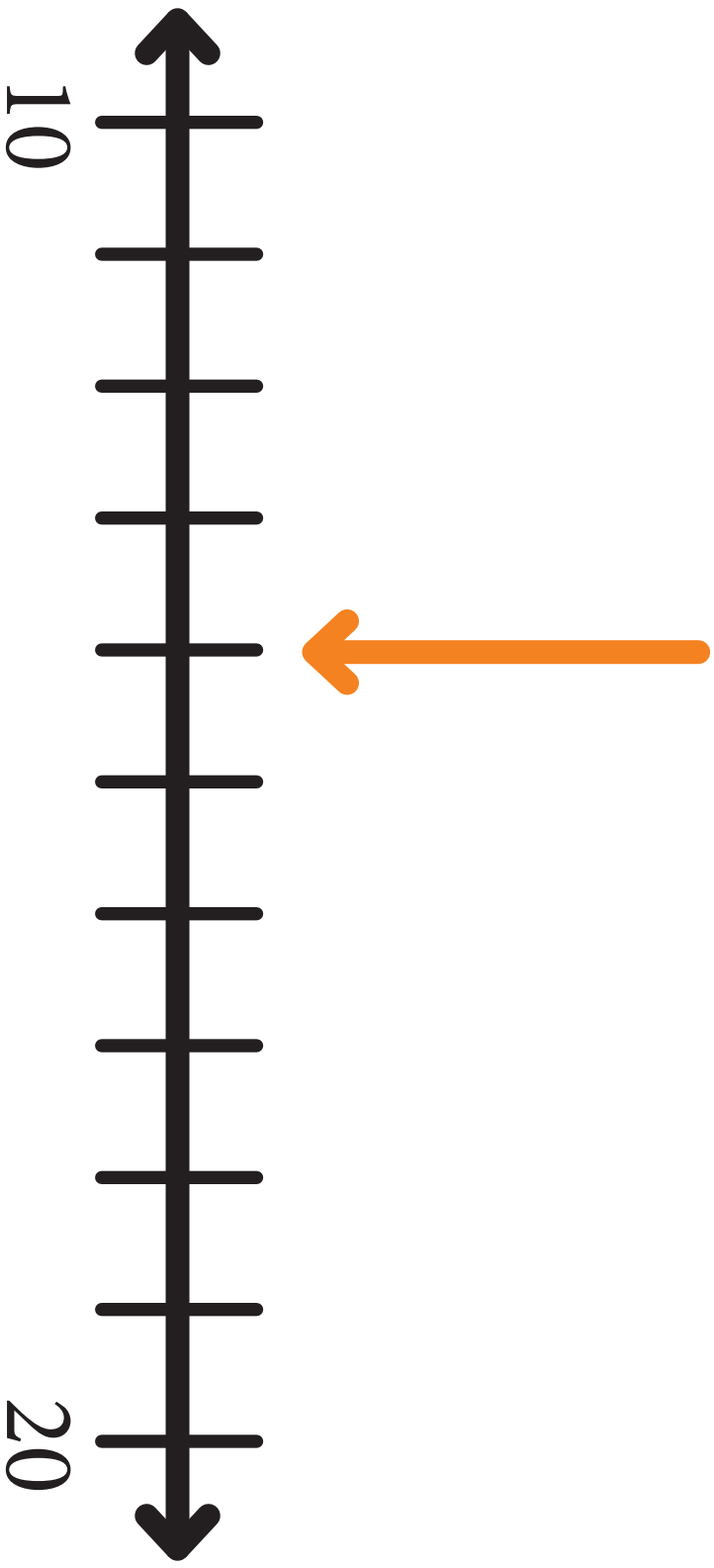
Correct response

7

For further diagnosis

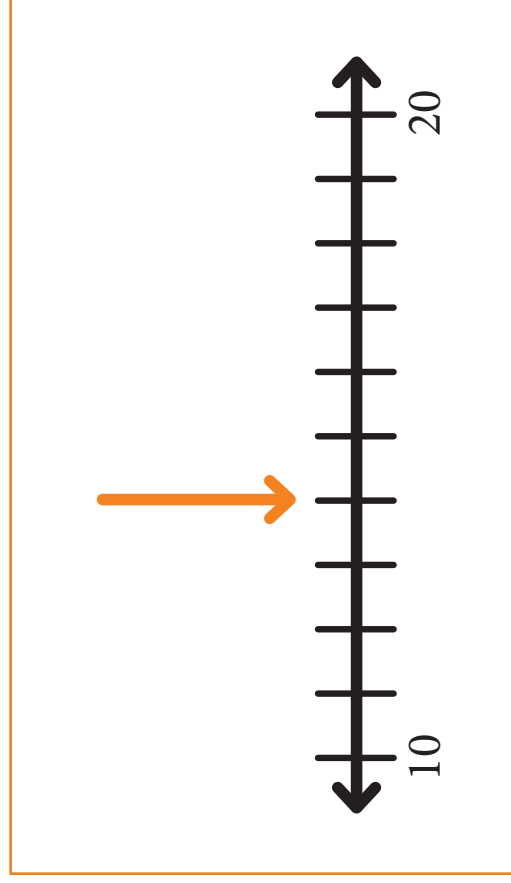
- Does the student count from 1?
- Does the student count back from 10?
- Does the student need to touch the line in order to keep track of the count?
- Does the student count on from 4?
- Does the student count the number of additional marks (3) and add $3 + 4$?
- Ask the student to identify other numbers that belong on the number line.

What Number is Below the Arrow?



What Number is Below the Arrow?

- Say, “This number line is missing some numbers. What number should be written below the arrow?”



Materials available

none

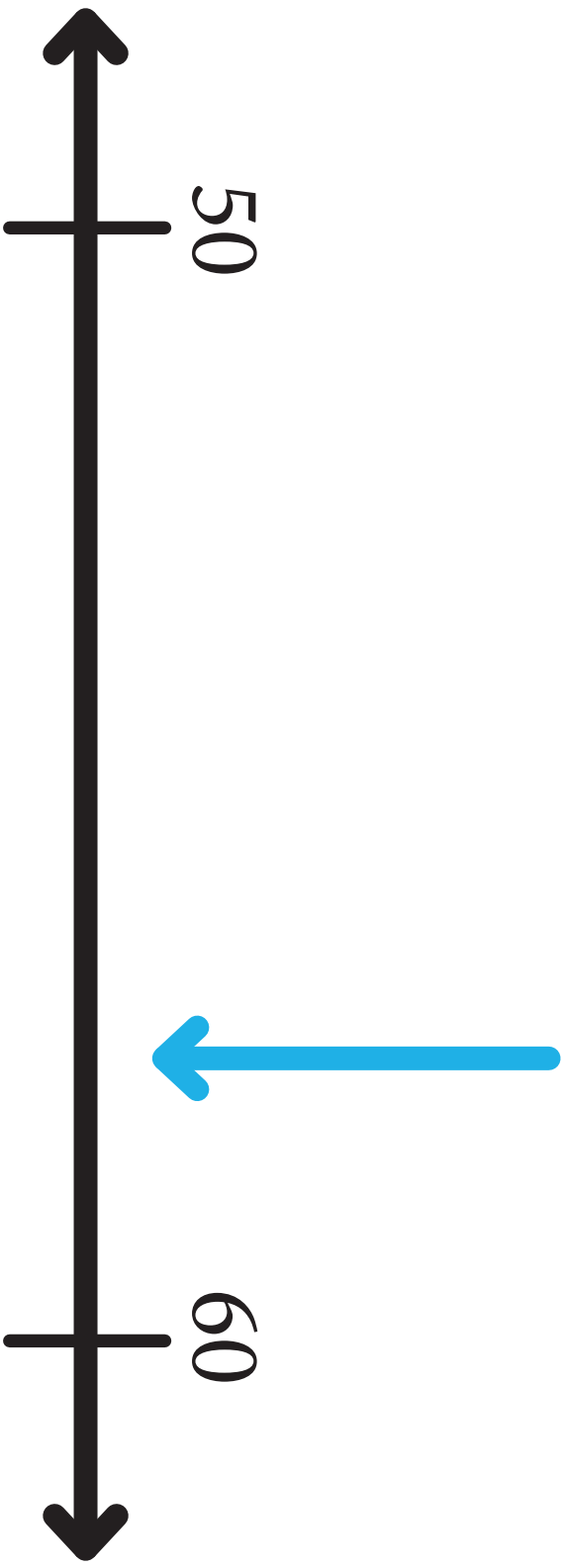
Correct response

14

For further diagnosis

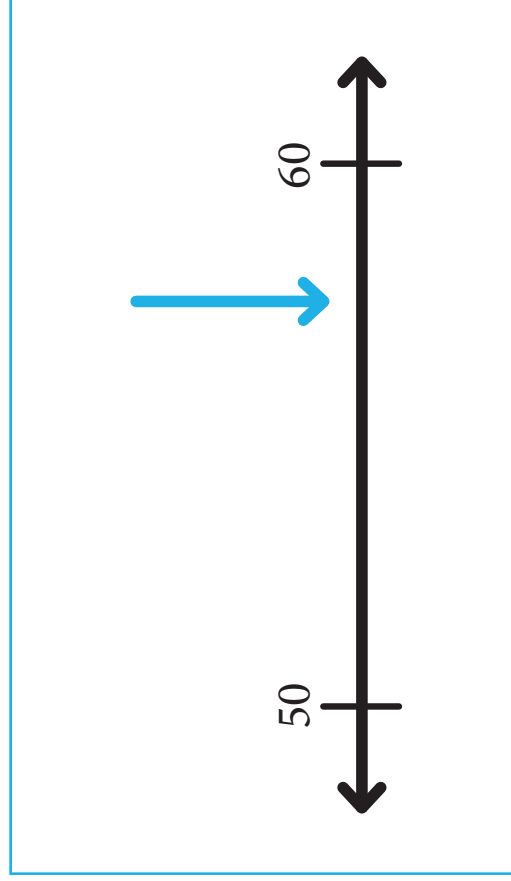
- Does the student count on from 10?
- Does the student use 15 (halfway between 10 and 20) as a landmark?
- Does the student count on to the arrow and then add 10 and four?
- Ask the student to identify other numbers that belong on the number line.

What Number is Below the Arrow?



What Number is Below the Arrow?

- Say, “This number line is missing some numbers. What number should be written below the arrow?”



Materials available

none

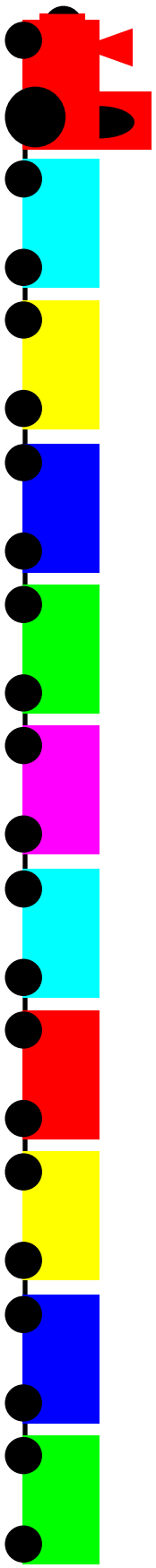
Correct response

57 or 58

For further diagnosis

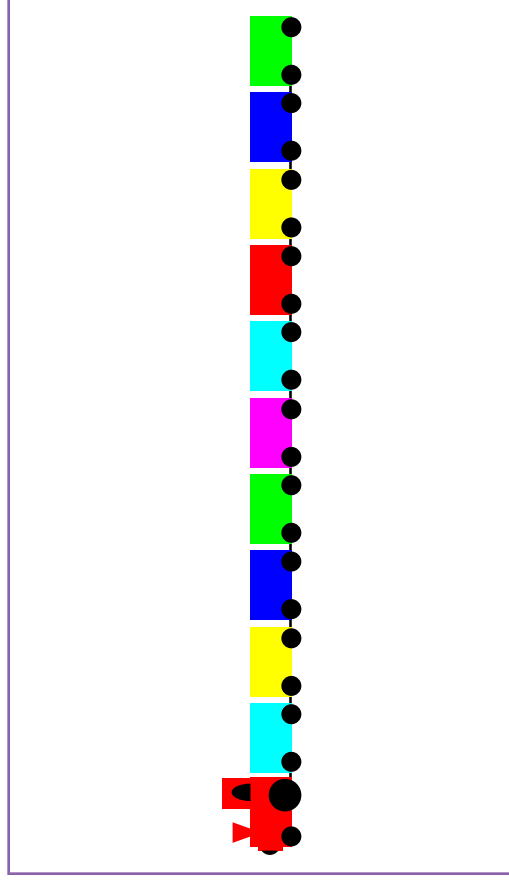
- Does the student use a finger width (erroneously) to count off the values?
- Does the student move down from 60 to find the value?
- Does the student use the halfway point (55) as a benchmark?
- Ask the student where other values belong (51, 55, 59).

Which Car is Ninth?



Which Car is Ninth?

- Say, “The light blue car is the first car in this train.” Point to the light blue car.
- “Which train car is in the ninth spot?”



Materials available

none

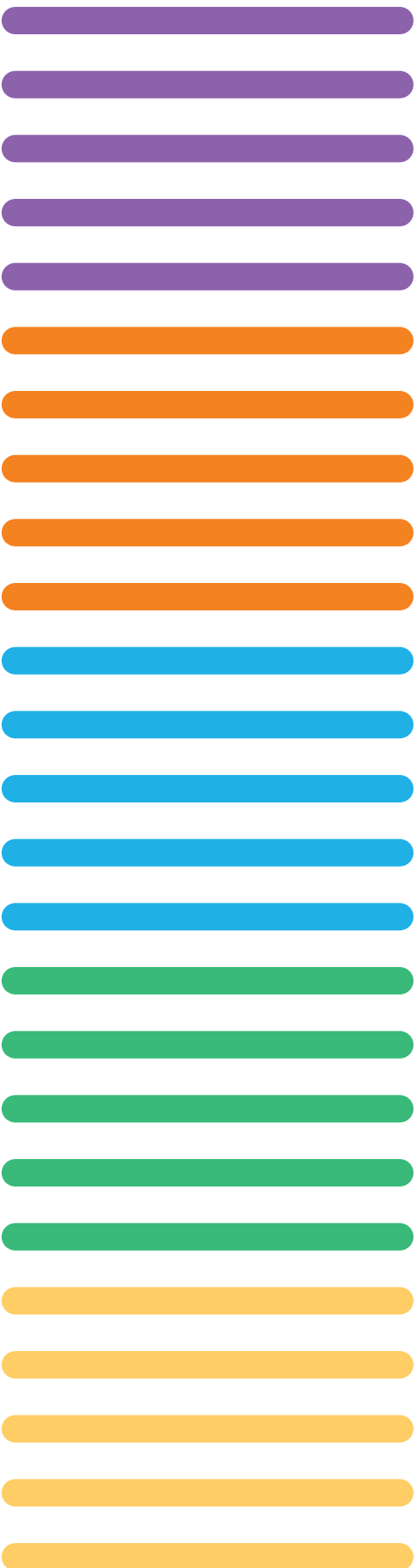
Correct response

Dark blue car (second from end)

For further diagnosis

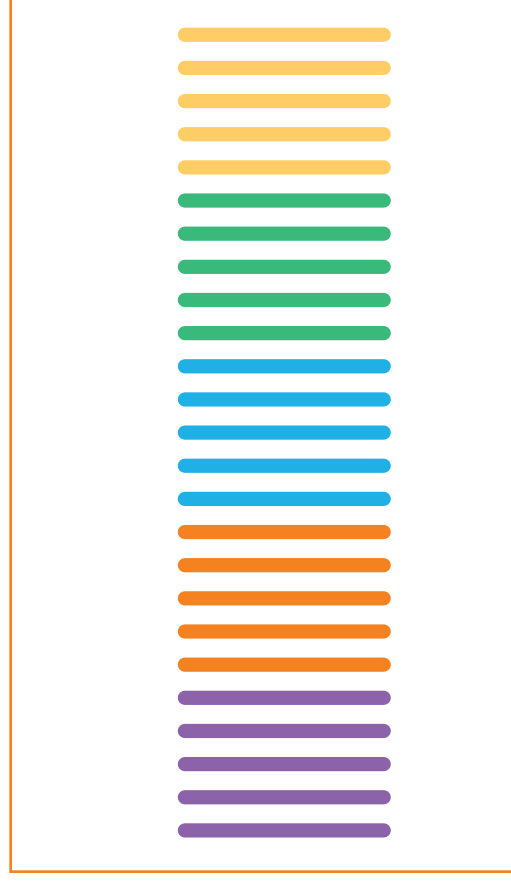
- Does the student count from left to right?
- Does the student count all the cars or stop when he or she reaches the ninth car?
- Does the student touch each car while counting?
- Ask the student to identify the cars associated with other ordinal values.

Which Bar is 20th?



Which Bar is 20th?

- Say, “Start counting at the left side. Which bar is 20th?”



Materials available

none

For further diagnosis

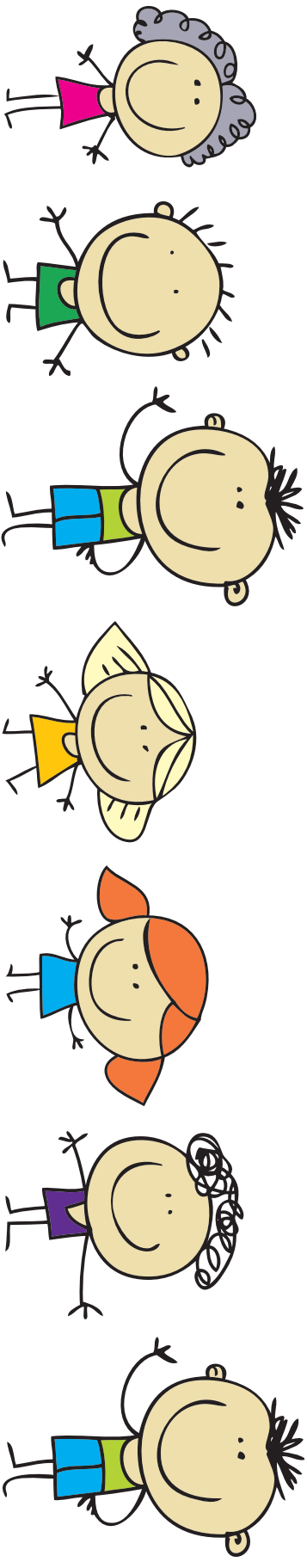
- Does the student count from left to right?
- Does the student touch each bar while counting?
- Does the student recognize and use groups of five?
- Ask the student to identify the bars associated with other ordinal values.

Correct response

Last green bar

Read and Find

Point to the **fifth** person in the line.

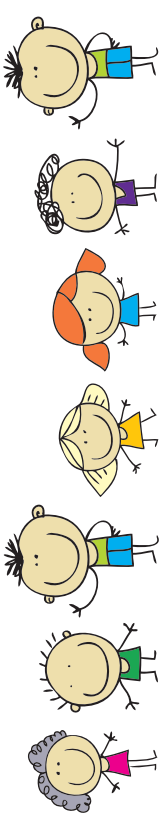


First

Read and Find

- Do NOT read the sentence aloud.
- Say, “Read this sentence and point to the correct child.”

Point to the **fifth** person in the line.



First

Materials available

none

Correct response

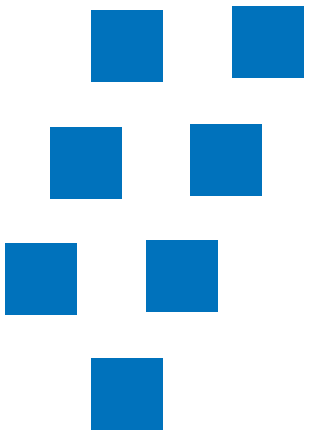
Girl with red hair and blue dress

For further diagnosis

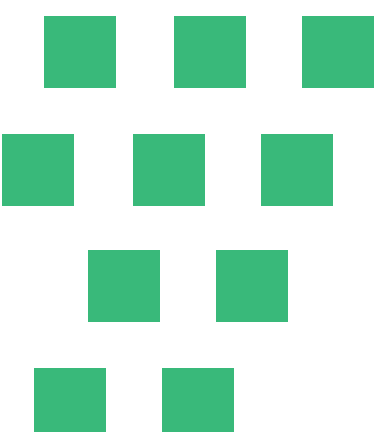
- Does the student correctly read the word “fifth?”
- Does the student start the count from the correct end?
- Write other ordinal words and ask the student to read them.
- Point to another child in the diagram and ask the student to tell you what position that child is in.

How Many More?

Randy




Nita



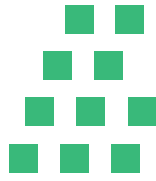
How Many More?

- Say, “Randy and Nita put cubes on this page.” Point to the side corresponding to each name.
- Say, “Randy wants to have the same amount as Nita. How many more cubes does Randy need?”

Randy



Nita



For further diagnosis

- Does the student build the block arrangements using linking cubes, then stack them to compare?
- Does the student place a finger on a blue square and another finger on a green one, continuing until all square have been tagged?
- Does the student count each side, then compare the numbers?
- Does the student see the shape of Randy's cubes contained in Nita's?

Materials available

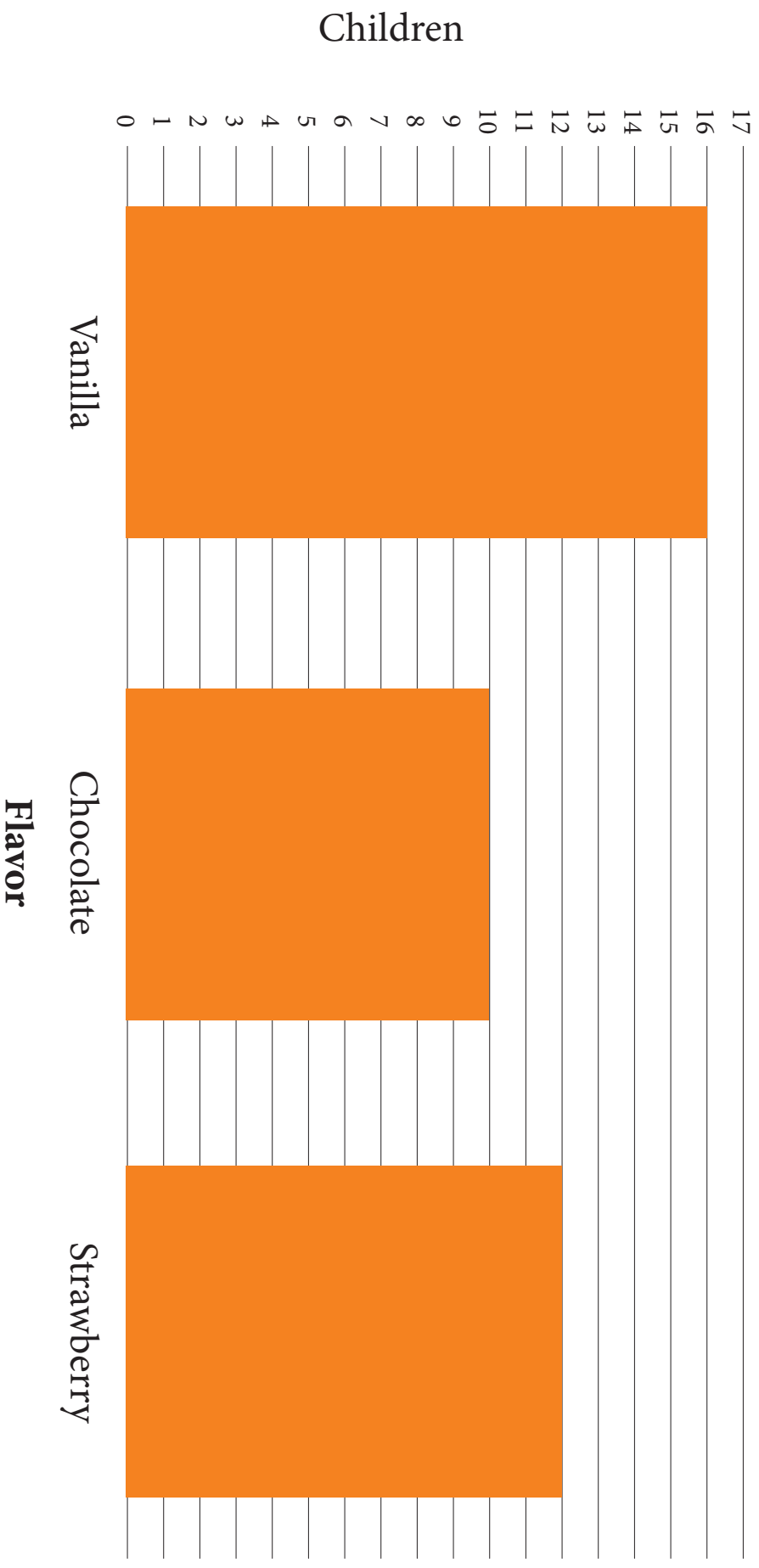


Correct response

3 (more cubes)

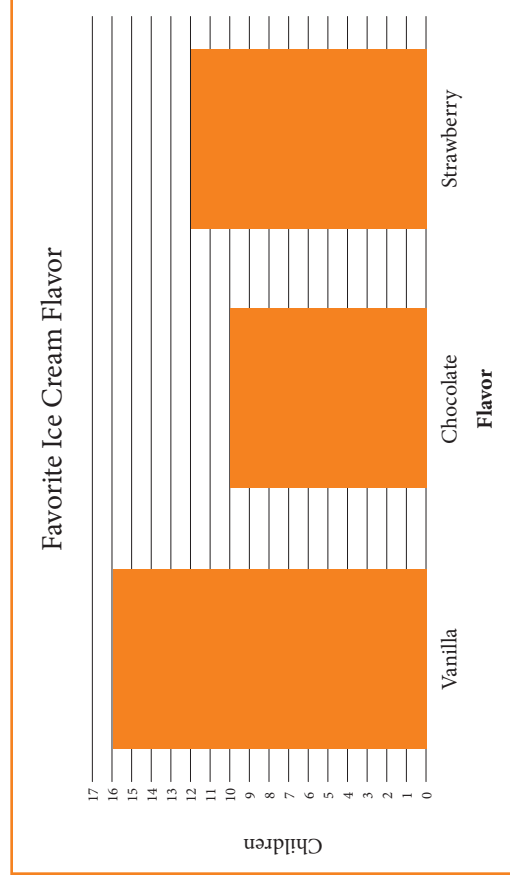
How Many More?

Favorite Ice Cream Flavor



How Many More?

- Say, “This graph shows the favorite flavor of ice cream for all the children in a first-grade class. How many more children like vanilla compared to strawberry?”



Materials available



Correct response

4 (more)

For further diagnosis

- Does the student use the correct bars for comparison?
- Does the student count up from the strawberry value?
- Does the student count back from the vanilla value?
- Does the student use the numbers on the scale and subtract?
- Ask the student to read the values for each flavor from the graph and build a cube stick for each.

How Many Points?

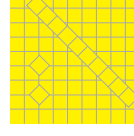
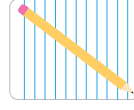
Falcons	Bears
32	43

How Many Points?

- Say, “This scoreboard shows the final score in a basketball game in which the Bears beat the Falcons. By how many points did the Bears win?”

Falcons	Bears
32	43

Materials available



For further diagnosis

- Does the student count up from 32 to 43?
- Does the student subtract?
- Ask the student how many more points the Falcons would need to score to make the game a tie.
- Change the Bears' score to a lower value and ask the same question (e.g., 38, 35).

Correct response

11 (points)

End of Concept Area 4

Concept Area 5

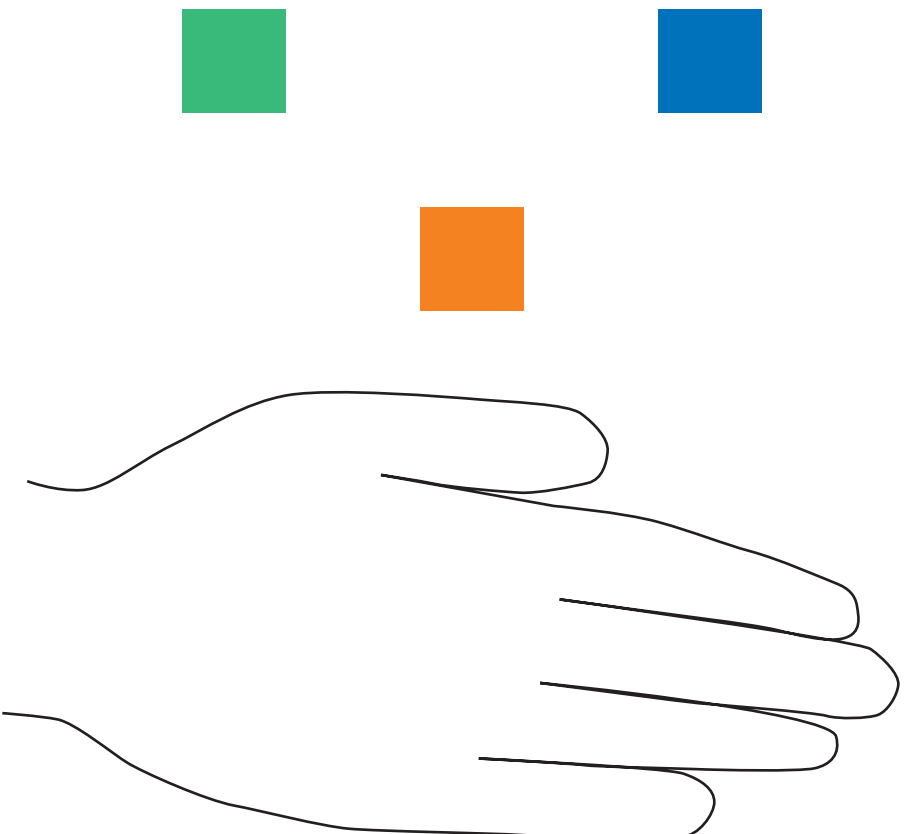
Competencies for Fluency With Number Combinations

Does the student understand and use these big ideas?

- Numbers can be composed and decomposed in multiple ways without changing their values
- Known addition number combinations can be used to determine unknown combinations
- The relationship between addition and subtraction can be used to determine unknown subtraction combinations

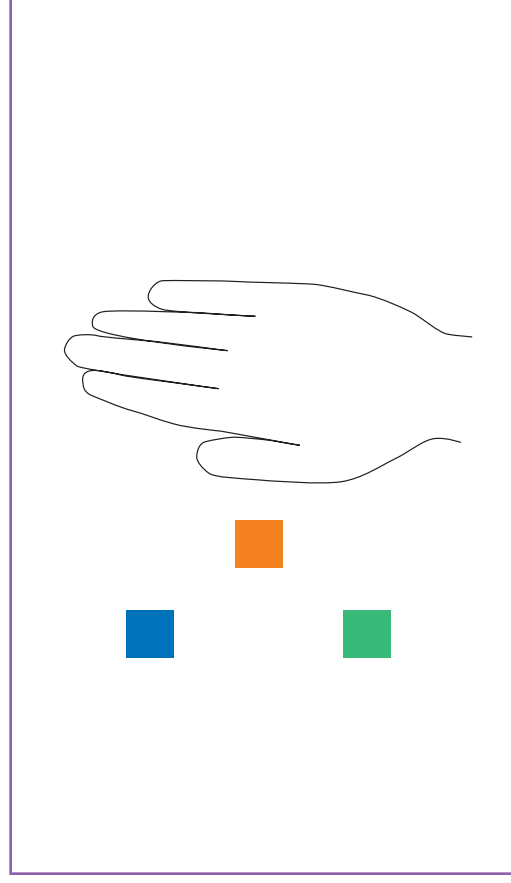
Learning goal	Performance level		
	A	B	C
5.1 Composes and decomposes numbers	Constructs partners, with objects, totals to 5	Constructs partners, with objects, totals to 10	
5.2 Knows “plus 1” is next counting word	Totals 2 to 5	Totals 6 to 9	
5.3 Knows addition combinations for doubles	Totals 2 to 5	Totals 6 to 9	Totals 11 to 18
5.4 Knows addition combinations for near doubles	Totals 2 to 5	Totals 6 to 9	Totals 11 to 18
5.5 Knows addition combinations based on 10		Totals equal to 10	Combinations with 9
5.6 Knows other addition combinations		Totals 6 to 9	Totals 11 to 18
5.7 Knows “minus 1” is previous counting word	Totals 2 to 5	Totals 6 to 9	
5.8 Knows subtraction combinations for doubles	Totals 2 to 5	Totals 6 to 9	Totals 11 to 18
5.9 Knows subtraction combinations for near doubles	Totals 2 to 5	Totals 6 to 9	Totals 11 to 18
5.10 Knows subtraction combinations based on 10		Totals equal to 10	Combinations with 9
5.11 Knows other subtraction combinations		Totals 6 to 9	Totals 11 to 18

What's Hidden?



What's Hidden?

- Say, “There are five squares on the page. Some of them are hidden under the hand. How many squares are hidden under the hand?”
- Act out the situation using cubes if the question is unclear to the student.



Materials available



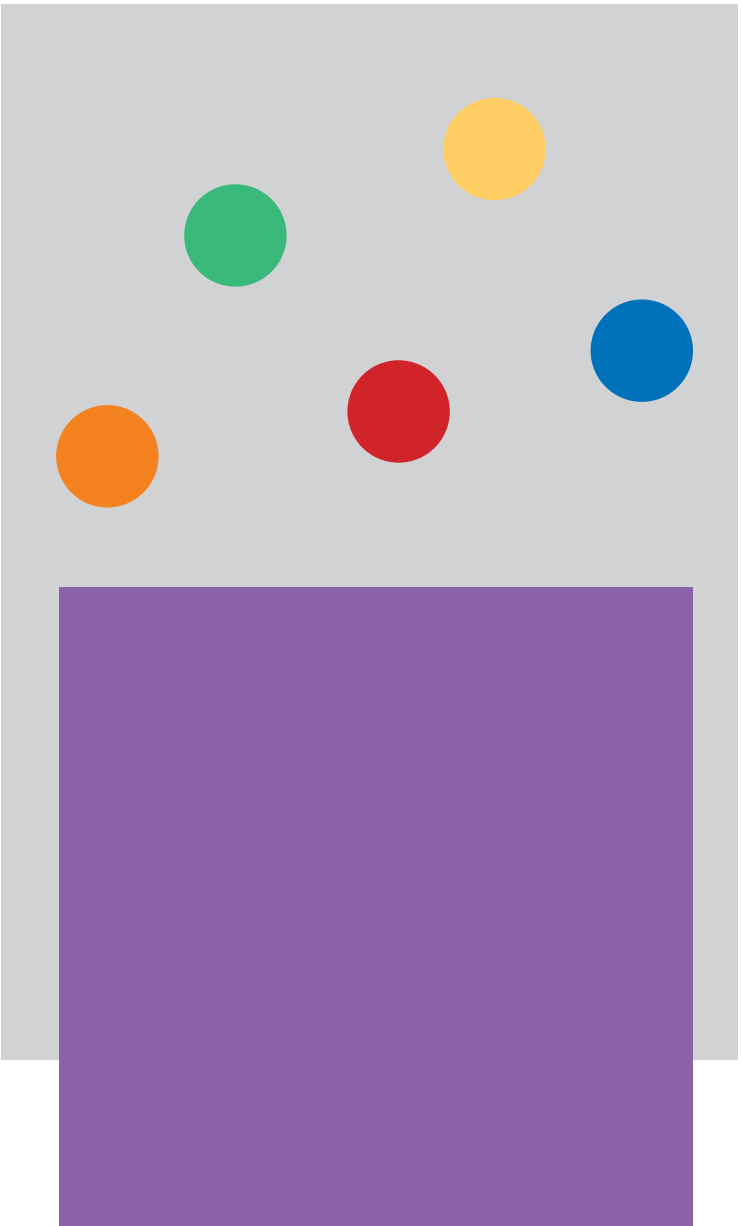
Correct response

2 (squares)

For further diagnosis

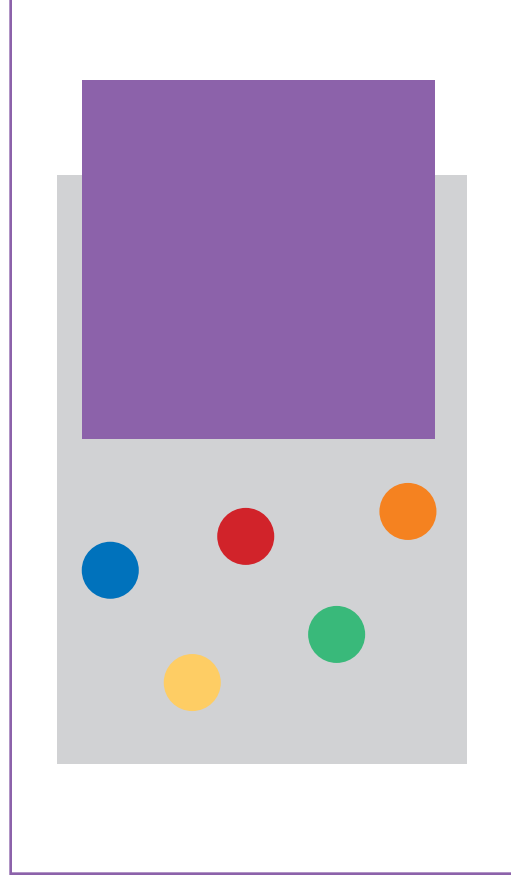
- Act out the scenario using real cubes under your hand.
- Use a smaller or larger number of cubes and different combinations to find the numbers the student is confident using. Hide or cover the items with your hand or a piece of paper.

What's Hidden?



What's Hidden?

- Say, “There are nine marbles on the gray mat. The purple paper is hiding some of the marbles. How many marbles are hidden under the paper?”
- Act out the situation using objects if the question is unclear to the student.



Materials available



Correct response

4 (marbles)

For further diagnosis

- Act out the scenario using real objects under a paper.
- Use a smaller or larger number of objects and different combinations to find the numbers the student is confident using. Hide or cover the items with your hand or a piece of paper.

What's the Answer?

3

+

1

=

What's the Answer?

- Say, “What is 3 plus 1?”
- You may also say,
 - “What is 3 and 1?” or
 - “What number is 1 more than 3?”

$$3 + 1 = \square$$

Materials available



Correct response

4

For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Ask the student to say the number after three.
- Reverse the order of the addends and ask again.
- Ask other +1 combinations with totals 2 to 5 with addends in either order
 - 1 + 1
 - 2 + 1
 - 4 + 1

What's the Answer?

6

+

1

=

What's the Answer?

- Say, “What is 6 plus 1?”
- You may also say,
 - “What is 6 and 1?” or
 - “What number is 1 more than 6?”

$$6 + 1 = \square$$

Materials available



Correct response

7

For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Ask the student to say the number after six.
- Reverse the order of the addends and ask again.
- Ask other +1 combinations with totals 6 to 9 with addends in either order
 - $5 + 1$
 - $7 + 1$
 - $8 + 1$

What's the Answer?

2

+

2

=

What's the Answer?

- Say, “What is 2 plus 2?”
- You may also say,
 - “What is 2 and 2?” or
 - “What number is 2 more than 2?”

$$2 + 2 = \square$$

Materials available



For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Ask other doubles combinations with totals 2 to 5
 - 1 + 1

Correct response

4

What's the Answer?

$$4 + 4 = \square$$

What's the Answer?

- Say, “What is 4 plus 4?”
- You may also say,
 - “What is 4 and 4?” or
 - “What number is 4 more than 4?”

$$4 + 4 = \square$$

Materials available



For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Ask other doubles combinations with totals 6 to 9
 - 3 + 3

Correct response

8

What's the Answer?

$$7 + 7 = \square$$

What's the Answer?

- Say, “What is 7 plus 7?”
- You may also say,
 - “What is 7 and 7?” or
 - “What number is 7 more than 7?”

$$7 + 7 = \square$$

For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Ask the student what 5 + 5 is. Does this help him or her find 7 + 7?
- Ask other doubles combinations with totals 11 to 18
 - 6 + 6
 - 8 + 8
 - 9 + 9

Materials available



Correct response

14

What's the Answer?

2

+

3

=

What's the Answer?

- Say, “What is 2 plus 3?”
- You may also say,
 - “What is 2 and 3?” or
 - “What number is 3 more than 2?”

$$2 + 3 = \square$$

Materials available



Correct response

5

For further diagnosis

- Does the student use direct modeling?
- Does the student count on from first? From larger?
- Reverse the order of the addends and ask again.
- Ask the student what $2 + 2$ is. Does this help him or her solve $2 + 3$?

What's the Answer?

5

+

4

=

What's the Answer?

- Say, “What is 5 plus 4?”
- You may also say,
 - “What is 5 and 4?” or
 - “What number is 4 more than 5?”

$$5 + 4 = \square$$

For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Reverse the order of the addends and ask again.
- Ask the student what $4 + 4$ is. Does this help him or her solve $5 + 4$?
- Ask other near doubles combinations with totals 6 to 9 with addends in either order
 - $3 + 4$

Materials available



Correct response

9

What's the Answer?

$$7 + 8 = \square$$

What's the Answer?

- Say, “What is 7 plus 8?”
- You may also say,
 - “What is 7 and 8?” or
 - “What number is 8 more than 7?”

For further diagnosis

- Does the student use direct modeling?
- Does the student count on from first? From larger?
- Reverse the order of the addends and ask again.
- Ask the student what $7 + 7$ is. Does this help him or her solve $7 + 8$?
- Ask other near doubles combinations with totals 11 to 18 with addends in either order

- $6 + 5$
- $7 + 6$
- $9 + 8$

$$7 + 8 = \square$$

Materials available



Correct response

15

What's the Answer?

3

+

7

=

What's the Answer?

- Say, “What is the answer?”
- You may also say, “What is 3 plus 7?”

$$3 + 7 = \square$$

For further diagnosis

- Does the student use direct modeling?
- Does the student count on from first? From larger?
- Reverse the order of the addends and ask again.
- Ask other combinations to 10 with addends in either order
 - 1 + 9
 - 2 + 8
 - 4 + 6
 - 5 + 5

Materials available



Correct response

10

What's the Answer?

$$9 + 5 = \square$$

What's the Answer?

- Say, “What is 9 plus 5?”
- You may also say,
 - “What is 9 and 5?” or
 - “What number is 5 more than 9?”

For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Reverse the order of the addends and ask again.
- Ask the student what $10 + 5$ is. Does this help him or her solve $9 + 5$?
- Ask other combinations with nine with addends in either order
 - $9 + 2$ $9 + 6$
 - $9 + 3$ $9 + 7$
 - $9 + 4$

$$9 + 5 = \square$$

Materials available



Correct response

14

What's the Answer?

$$4 + 2 = \square$$

What's the Answer?

- Say, “What is 4 plus 2?”
- You may also say,
 - “What is 4 and 2?” or
 - “What number is 2 more than 4?”


For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Reverse the order of the addends and ask again.
- Ask the student what combination near $4 + 2$ he or she knows. Ask if this combination helps him or her to find this answer.
- Ask other combinations within 10 with addends in either order

- $2 + 5$ $2 + 7$
- $2 + 6$ $3 + 5$
- $3 + 6$

$$4 + 2 = \square$$

Materials available



Correct response

6

What's the Answer?

8

+

4

=

What's the Answer?

- Say, “What is the answer?”
- You may also say, “What is 8 plus 4?”

For further diagnosis

- Does the student use direct modeling?
- Does the student count on?
- Reverse the order of the addends and ask again.
- Ask the student what combination near $8 + 4$ he or she knows. Ask if this combination helps him or her to find this answer.
- Ask other combinations with totals 11 to 18 with addends in either order

- $3 + 8$
- $4 + 7$
- $5 + 7$
- $5 + 8$
- $6 + 8$

$$8 + 4 = \square$$

Materials available



Correct response

12

What's the Answer?

3

-

1

=

What's the Answer?

- Say, “What is 3 minus 1?”
- You may also say,
 - “What is 3 take away 1?” or
 - “What number is 1 less than 3?”

$$3 - 1 = \square$$

Materials available



For further diagnosis

- Does the student use direct modeling?
- Does the student count back?
- Ask the student to say the number before three.
- Ask other -1 combinations with totals 2 to 5
 - $4 - 1$
 - $2 - 1$

Correct response

2

What's the Answer?

8

-

1

=

What's the Answer?

- Say, “What is 8 minus 1?”
- You may also say,
 - “What is 8 take away 1?” or
 - “What number is 1 less than 8?”

$$8 - 1 = \square$$

Materials available



For further diagnosis

- Does the student use direct modeling?
- Does the student count back?
- Ask the student to say the number before eight.
- Ask other -1 combinations with totals 6 to 9
 - $9 - 1$
 - $7 - 1$
 - $6 - 1$

Correct response

7

What's the Answer?

4

-

2

=

What's the Answer?

- Say, “What is 4 minus 2?”
- You may also say,
 - “What is 4 take away 2?” or
 - “What number is 2 less than 4?”

$$4 - 2 = \square$$

Materials available



Correct response

2

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from four or up from two?
- Ask the student the related missing addend problem ($2 + \text{what is } 4?$).
- Ask other subtraction combinations for doubles with totals 2 to 5
 - $2 - 1$

What's the Answer?

$$6 - 3 = \square$$

What's the Answer?

- Say, “What is 6 minus 3?”
- You may also say,
 - “What is 6 take away 3?” or
 - “What number is 3 less than 6?”

$$6 - 3 = \square$$

Materials available



Correct response

3

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from six or up from three?
- Ask the student the related missing addend problem (3 + what is 6?).
- Ask other subtraction combinations for doubles with totals 6 to 9
 - 8 – 4

What's the Answer?

$$16 - 8 = \square$$

What's the Answer?

- Say, “What is 16 minus 8?”
- You may also say,
 - “What is 16 take away 8?” or
 - “What number is 8 less than 16?”

$$16 - 8 = \square$$

Materials available



Correct response

8

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from 16 or up from eight?
- Ask the student the related missing addend problem (8 + what is 16?).
- Ask other subtraction combinations for doubles with totals 11 to 18
 - 12 – 6
 - 14 – 7
 - 18 – 9

What's the Answer?

5

-

3

=

What's the Answer?

- Say, “What is 5 minus 3?”
- You may also say,
 - “What is 5 take away 3?” or
 - “What number is 3 less than 5?”

$$5 - 3 = \square$$

Materials available



Correct response

2

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from five or up from three?
- Ask the student the related missing addend problem (3 + what is 5?).
- Ask other subtraction combinations for doubles with totals 2 to 5
 - 5 – 2

What's the Answer?

$$7 - 4 = \square$$

What's the Answer?

- Say, “What is the answer?”
- You may also say, “What is 7 minus 4?”

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from seven or up from four?
- Ask the student the related missing addend problem (4 + what is 7?).
- Ask other subtraction combinations for near doubles with totals 6 to 9

- 5 – 3
- 5 – 2
- 7 – 3
- 9 – 4
- 9 – 5

$$7 - 4 = \square$$

Materials available



Correct response

3

What's the Answer?

$$13 - 6 =$$

What's the Answer?

- Say, “What is 13 minus 6?”
- You may also say,
 - “What is 13 take away 6?” or
 - “What number is 6 less than 13?”

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from 13 or up from six?
- Ask the student the related missing addend problem (6 + what is 13?).
- Ask 13 – 7.
- Ask other subtraction combinations for near doubles with totals 11 to 18
 - 11 – 5 15 – 8
 - 11 – 6 17 – 8
 - 15 – 7 17 – 9

$$13 - 6 = \square$$

Materials available



Correct response

7

What's the Answer?

$$10 - 8 =$$

What's the Answer?

- Say, “What is the answer?”
- You may also say, “What is 10 minus 8?”


For further diagnosis

- Does the student use direct modeling?
- Does the student count back from 10 or up from eight?
- Ask the student the related missing addend problem (8 + what is 10?).
- Ask 10 – 2.
- Ask other subtraction combinations for sums to 10

- 10 – 1
- 10 – 9
- 10 – 3
- 10 – 7
- 10 – 4
- 10 – 6
- 10 – 5

$$10 - 8 = \square$$

Materials available



Correct response

2

What's the Answer?

$$15 - 9 =$$

What's the Answer?

- Say, “What is 15 minus 9?”
- You may also say,
 - “What is 15 take away 9?” or
 - “What number is 9 less than 15?”

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from 15 or up from nine?
- Ask the student the related missing addend problem (9 + what is 15?).
- Ask 15 – 10. Ask how this can help solve the problem.
- Ask other subtraction combinations for combinations with nine

- 11 – 2 11 – 9 14 – 5
- 12 – 3 12 – 9 14 – 9
- 13 – 4 13 – 9
- 16 – 7 16 – 9

$$15 - 9 = \square$$

Materials available



Correct response

6

What's the Answer?

$$9 - 6 = \square$$

What's the Answer?

- Say, “What is 9 minus 6?”
- You may also say,
 - “What is 9 take away 6?” or
 - “What number is 6 less than 9?”

$$9 - 6 = \square$$

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from nine or on from six?
- Ask the student the related missing addend problem (6 + what is 9?).
- Ask 9 – 3.
- Ask other subtraction combinations with totals 6 to 9

- 6 – 2 6 – 4 8 – 3
- 7 – 5 7 – 2 8 – 5
- 8 – 2 8 – 6
- 9 – 2 9 – 7

Materials available



Correct response

3

What's the Answer?

$$12 - 7 = \square$$

What's the Answer?

- Say, “What is 12 minus 7?”
- You may also say,
 - “What is 12 take away 7?” or
 - “What number is 7 less than 12?”

For further diagnosis

- Does the student use direct modeling?
- Does the student count back from 12 or up from seven?
- Ask the student the related missing addend problem (7 + what is 12?).
- Ask 12 – 5.
- Ask other subtraction combinations with totals 11 to 18
 - 11 – 3 11 – 8 14 – 6
 - 11 – 4 11 – 7 14 – 8
 - 12 – 4 12 – 8
 - 13 – 5 13 – 8

$$12 - 7 = \square$$

Materials available



Correct response

5

End of Concept Area 5

Concept Area 6

Competencies for Properties and Symbols

Does the student understand and use these big ideas?

- Number relationships can be expressed in both words and symbols
- Addition and subtraction are connected to counting forward and backward
- Using properties of numbers and operations can simplify calculations

Learning goal	Performance level		
	A	B	C
6.1 Translates between word problems and number sentences	<i>JRU, totals 2 to 9</i>	SCU, totals 11 to 18	JSU, totals 11 to 18
6.2 Identifies the connection between addition and subtraction and counting forward and backward	Connects adding to counting on	<i>Connects subtracting to counting back</i>	
6.3 Compares numbers using symbols	<i>Using =</i>	<i>Using <, ></i>	<i>Produces symbol</i>
6.4 Recognizes and uses properties of addition		Commutative property	<i>Associative property to add 3 single-digit numbers</i>
6.5 Recognizes addition-subtraction complement and inverse principle	<i>Inverse principle, single-digit numbers in a context</i>	<i>Complement principle in a context</i>	<i>Inverse principle in symbols</i>

JRU = Join, result known
 SCU = Separate, change unknown
 JSU = Join, start unknown

Which Number Sentence?

Mike has 4 pennies. His mom gave him 3 more pennies. Now how many pennies does Mike have?

$$4 - 3 = \square$$

$$7 + 3 = \square$$

$$4 + 3 = \square$$

Which Number Sentence?

- Say, “I will read you a number story. Tell me which number sentence matches it.”
- Repeat if requested.
- If necessary, explain that we do not need to know the answer to the question, only which number sentence matches the word sentence.

For further diagnosis

- Ask the student to model the situation with objects, and then ask which sentence matches.
- Does the student know the plus, minus, and equal symbols?
- Does the student accurately read numbers?
- Ask the student to tell a number story for each of the number sentences.

Mike has 4 pennies. His mom gave him 3 more pennies. Now how many pennies does Mike have?

$$4 - 3 = \square$$

$$7 + 3 = \square$$

$$4 + 3 = \square$$

Materials available



Correct response

$$4 + 3 = \square \text{ (last option)}$$

Which Number Sentence?

There were 15 brownies on a plate. Jenna took some brownies. Now there are 6 brownies on the plate.

$$15 + 6 = \square$$

$$15 - \square = 6$$

$$15 + \square = 6$$

How many brownies did Jenna take?

Which Number Sentence?

- Say, “I will read you a number story. Tell me which number sentence matches it.”
- Repeat if requested.
- If necessary, explain that we do not need to know the answer to the question, only which number sentence matches the word sentence.

For further diagnosis

- Ask the student to model the situation with objects, and then ask which sentence matches.
- Does the student know the plus, minus, and equal symbols?
- Does the student accurately read numbers?
- Ask the student to tell a number story for each of the number sentences.

There were 15 brownies on a plate. Jenna took some brownies. Now there are 6 brownies on the plate. How many brownies did Jenna take?

15 + 6 = □
 15 - □ = 6
 15 + □ = 6

Materials available



Correct response

15 - □ = 6 (middle option)

Which Number Sentence?

There were some snakes on a rock. Twelve more snakes came to the rock. Now there are 18 snakes on the rock. How many snakes were on the rock to start?

$$12 + 18 = \square$$

$$\square + 12 = 18$$

$$\square - 12 = 18$$

Which Number Sentence?

- Say, “I will read you a number story. Tell me which number sentence matches it.”
- Repeat if requested.
- If necessary, explain that we do not need to know the answer to the question, only which number sentence matches the word sentence.

For further diagnosis

- Ask the student to model the situation with objects, and then ask which sentence matches.
- Does the student know the plus, minus, and equal symbols?
- Does the student accurately read numbers?
- Ask the student to tell a number story for each of the number sentences.

There were some snakes on a rock. Twelve more snakes came to the rock. Now there are 18 snakes on the rock. How many snakes were on the rock to start?

$$12 + 18 = \square$$

$$\square + 12 = 18$$

$$\square - 12 = 18$$

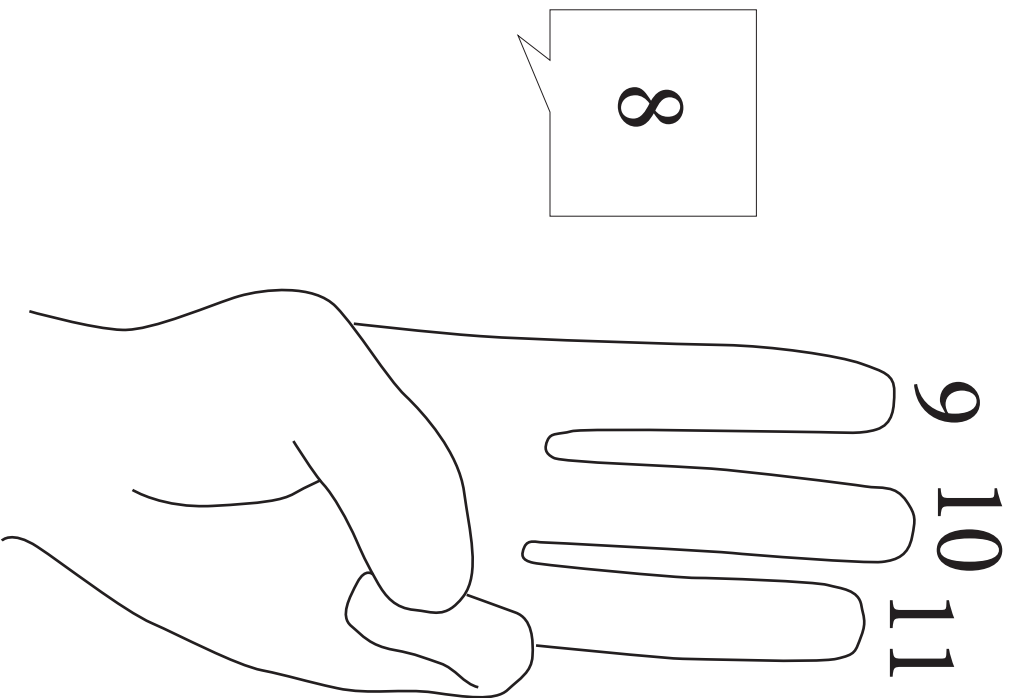
Materials available



Correct response

$$\square + 12 = 18 \text{ (middle option)}$$

Which Problem Matches?



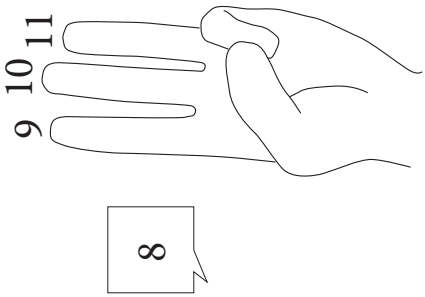
$$8 + 3 = \square$$

$$8 + 11 = \square$$

$$9 + 3 = \square$$

Which Problem Matches?

- Say, “My friend was working on one of these math problems. I heard him say ‘8 (pause), 9, 10, 11’ using his fingers like this (act this out). Which problem was he solving?”



8 + 3 =

8 + 11 =

9 + 3 =

Materials available

none

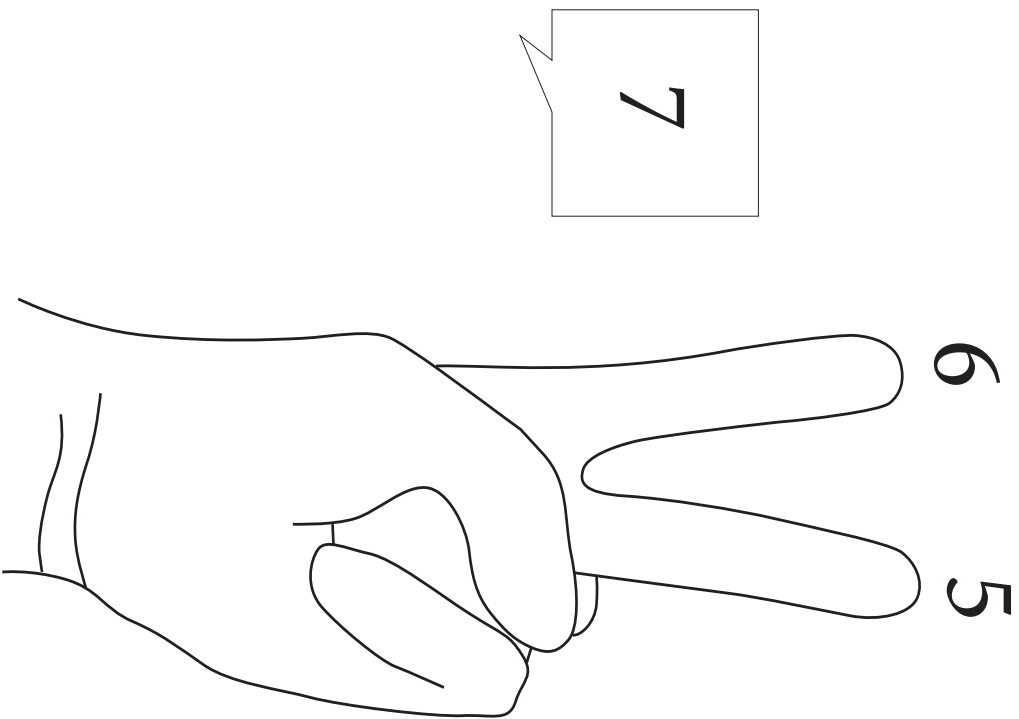
Correct response

8 + 3 = (first option)

For further diagnosis

- Have the student act out the counting scenario.
- Cover up the picture and ask the student to use a counting strategy for each of the number sentences.
- Ask the student which operation (addition or subtraction) is connected to counting up.

Which Problem Matches?



$$7 + 2 = \square$$

$$7 - 6 = \square$$

$$7 - 2 = \square$$

Which Problem Matches?

- Say, “My other friend was working on one of these math problems. I heard her say ‘7 (pause), 6, 5’ using her fingers like this (act this out). Which problem was she solving?”

6 5

7

$7 + 2 = \square$

$7 - 6 = \square$

$7 - 2 = \square$

Materials available

none

Correct response

$7 - 2 = \square$ (last option)

For further diagnosis

- Have the student act out the counting scenario.
- Cover up the picture and ask the student to use a counting strategy for each of the number sentences.
- Ask the student which operation (addition or subtraction) is connected to counting back.

Which Symbol Belongs Here?

Which symbol belongs in the square to make the statement true?

$<$ $=$ $>$

$$4 + 4 \square 1 + 7$$

Which Symbol Belongs Here?

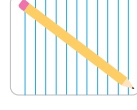
- Say, “Which symbol belongs in the square to make the statement true?”

Which symbol belongs in the square to make the statement true?

< = >

$4 + 4$ $1 + 7$

Materials available



For further diagnosis

- Ask the student to say the name of each symbol.
- Ask the student what the total is for each side.
- Pose a similar problem with a single number on each side.

Correct response

= (middle option)

Which Symbol Belongs Here?

Which symbol belongs in the square to make the statement true?

$<$ $=$ $>$

43 21

Which Symbol Belongs Here?

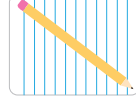
- Say, “Which symbol belongs in the square to make this statement true?”

Which symbol belongs in the square to make the statement true?

< = >

43 21

Materials available



For further diagnosis

- Ask the student to say the name of each symbol.
- Ask the student to point to the greater number.
- Ask the student to locate these two numbers on a number line.
- Pose a similar problem with smaller numbers.

Correct response

greater than > (last option)

What Symbol?

What symbol could you write in the square to make this a true number sentence?

$$12 \square 50$$

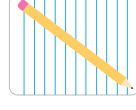
What Symbol?

- Say, “What symbol could you write in the square to make this a true number sentence? Write the numbers and the symbol on your paper.”

What symbol could you write in the square to make this a true number sentence?

$$12 \square 50$$

Materials available



For further diagnosis

- Ask the student to point to the larger number.
- Write each symbol ($<$, $>$, $=$) and ask the student to read each one.

Correct response

Writes “ $12 < 50$ ” or “ $50 > 12$ ”

Which Number Sentence?

One of these number sentences is true.

Which one is true?

a) $9 + 5 = 9 - 5$

b) $9 + 5 = 9 + 5 - 5$

c) $9 + 5 = 5 + 9$

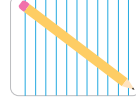
Which Number Sentence?

- Say, “One of these number sentences is true and the others are false. Which one is true?”

One of these number sentences is true.
Which one is true?

- a) $9 + 5 = 9 - 5$
- b) $9 + 5 = 9 + 5 - 5$
- c) $9 + 5 = 5 + 9$

Materials available



Correct response

- c) $9 + 5 = 5 + 9$

For further diagnosis

- Ask the student to explain the reasoning for his or her answer choice.
- Does he or she need to compute each value in order to answer, or does he or she use numerical reasoning?
- Ask the student to read each number sentence aloud.
- Ask the student to write or say the fact family for $9 + 5$.
- Ask the student to model $9 + 5$ and $5 + 9$. Does the student realize both sums are the same?
- Ask the same question using smaller numbers.

Find Friendly Combinations

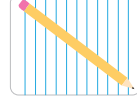
$$7 + 9 + 1$$

Find Friendly Combinations

- Say, “I want to add this set of numbers quickly in my head. What combination could I add first that is ‘friendly?’”

$$7 + 9 + 1$$

Materials available



For further diagnosis

- Ask the student to look for a combination that makes 10.
- Ask the student what combinations are “friendly” or easy to calculate.
- Ask the student if changing the order of the numbers in the problem would change the total.

Correct response

Add $9 + 1 = 10$ first (or another response that accurately uses a rearrangement of the numbers to simplify calculation)

How Many Pennies?

I had 8 pennies in my pocket. My dad gave me 5 more pennies, and then I gave 5 pennies to my little sister. How many pennies do I have now?

How Many Pennies?

- Read the problem aloud.

I had 8 pennies in my pocket. My dad gave me 5 more pennies, and then I gave 5 pennies to my little sister. How many pennies do I have now?

Materials available

none

For further diagnosis

- Does the student accurately use numerical reasoning to determine the answer?
- Ask the student to model this problem with objects.
- Ask the student how he or she could “undo” giving five pennies.
- Ask the student if the problem has the same result if the dad gave three more pennies and three pennies are given to the sister.

Correct response

8 (pennies)

Finish the Fact Family

$$5 + 6 = 11$$

$$11 - 5 = 6$$

Finish the Fact Family

- Say, “There are two related number sentences on the page. What are some other number sentences that are part of this ‘fact family?’”

$$5 + 6 = 11 \qquad 11 - 5 = 6$$

Materials available



Correct response

$$6 + 5 = 11 \text{ and } 11 - 6 = 5$$

For further diagnosis

- Ask the student how the two number sentences are related.
- Ask the student to build a representation of $5 + 6 = 11$ using sets of cubes in two colors. Have the student use the blocks to show the second number sentence. Ask if there are other number sentences that can be shown using the same collection of blocks.

True or False?

$$8 + \square - \square = 8$$

True or False?

- Say, “If we put the same number in each box, would this number sentence be true or false?”

$$8 + \square - \square = 8$$

Materials available



For further diagnosis

- Does the student understand the meaning of the number sentence?
- Ask the student to explain his or her understanding of the equal sign.
- Ask the student to place the number “1” in the box and work out the problem. Now place the number “2” in the box. Continue until the student sees the pattern.
- Have the student model the problem using materials.

Correct response

True

End of Content Area 6

Concept Area 7

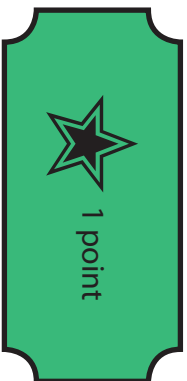
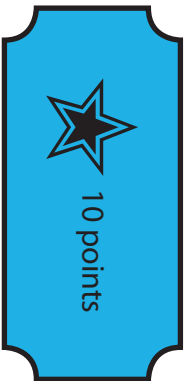
Competencies for Place Value

Does the student understand and use these big ideas?

- Items can be grouped or ungrouped according to place value units
- The position of a digit (in a multidigit number) indicates its value
- Computing with multidigit numbers can involve regrouping based on place value

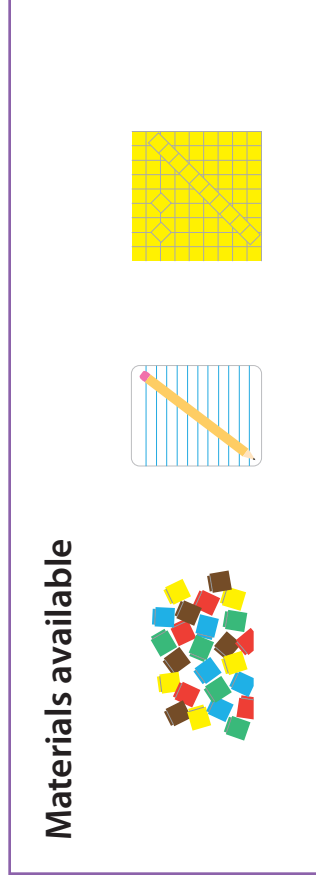
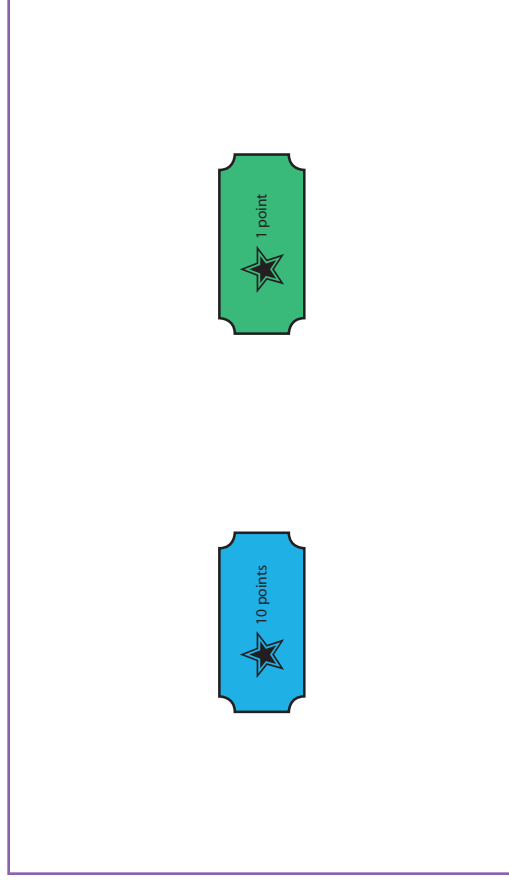
Learning goal	Performance level		
	A	B	C
7.1 Recognizes base-ten equivalents	<i>10 ones = 1 ten</i>	<i>10 tens = 100 ones = 1 hundred</i>	<i>10 hundreds = 1,000</i>
7.2 Translates among place value models, count words, numerals	Teens	2-digit numbers	3-digit numbers
7.3 Reads and writes multidigit numbers meaningfully	To 20	2-digit numbers	3-digit numbers
7.4 Decomposes a larger unit into smaller units by place value	To 30	2-digit numbers	3-digit numbers
7.5 Adds multidigit whole numbers	<i>2-digit numbers, without regrouping</i>	<i>2-digit numbers, with regrouping</i>	<i>Explains a renaming algorithm for addition</i>
7.6 Subtracts multidigit whole numbers	<i>2-digit numbers, without regrouping</i>	<i>2-digit numbers, with regrouping</i>	<i>Explains a renaming algorithm for subtraction</i>

How Many To Match?



How Many To Match?

- Say, “At the carnival you can win blue tickets or green tickets. Blue tickets are worth 10 points, and green tickets are worth 1 point. How many green tickets would you need to trade for one blue ticket?”



For further diagnosis

- Ask the student to build sticks of cubes to represent each color of ticket.
- Ask the student to identify the base-ten blocks that correspond to the two tickets.

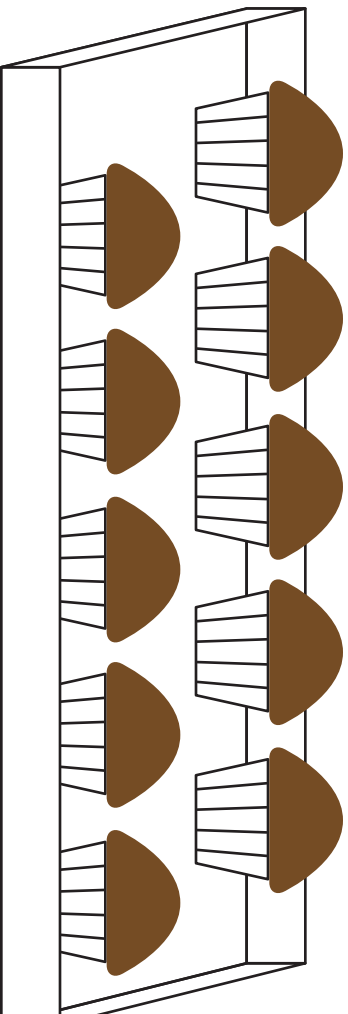
Correct response

10 (green tickets)

Buying Cupcakes

The bakery sells boxes of cupcakes with 10 in each box.

How many boxes should we buy if we need 100 cupcakes?

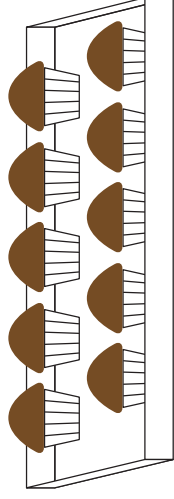


Buying Cupcakes

- Read the problem aloud.

The bakery sells boxes of cupcakes with 10 in each box.

How many boxes should we buy if we need 100 cupcakes?



Materials available



For further diagnosis

- Does the student count by tens to 100?
- Does the student use base-ten blocks to solve the problem?
- Ask the student to identify the base-ten blocks that could represent the situation in the problem.

Correct response

10 (boxes)

Pencil Boxes

Pencils come in boxes with 100 in each box.
We need 1,000 pencils. How many boxes of
pencils should we buy?



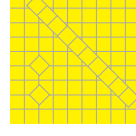
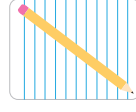
Pencil Boxes

- Read the problem aloud.

Pencils come in boxes with 100 in each box.
We need 1,000 pencils. How many boxes of pencils should we buy?



Materials available



For further diagnosis

- Does the student count by hundreds to 1,000?
- Does the student use base-ten blocks to solve the problem?
- Ask the student to identify the base-ten blocks that could represent the situation in the problem.

Correct response

10 (boxes)

Show This Amount

Fourteen

Show This Amount

- Place base-ten blocks and linking cubes within easy reach.
- Say, **“Show this amount using these materials. Use the materials to show me the tens and the ones.”**

For further diagnosis

- Does the student recognize that 14 is the combination of 1 ten and 4 ones?
- Ask the student to write the number 14.
- Ask the student to take out 14 individual cubes and then group them to form tens.

Fourteen

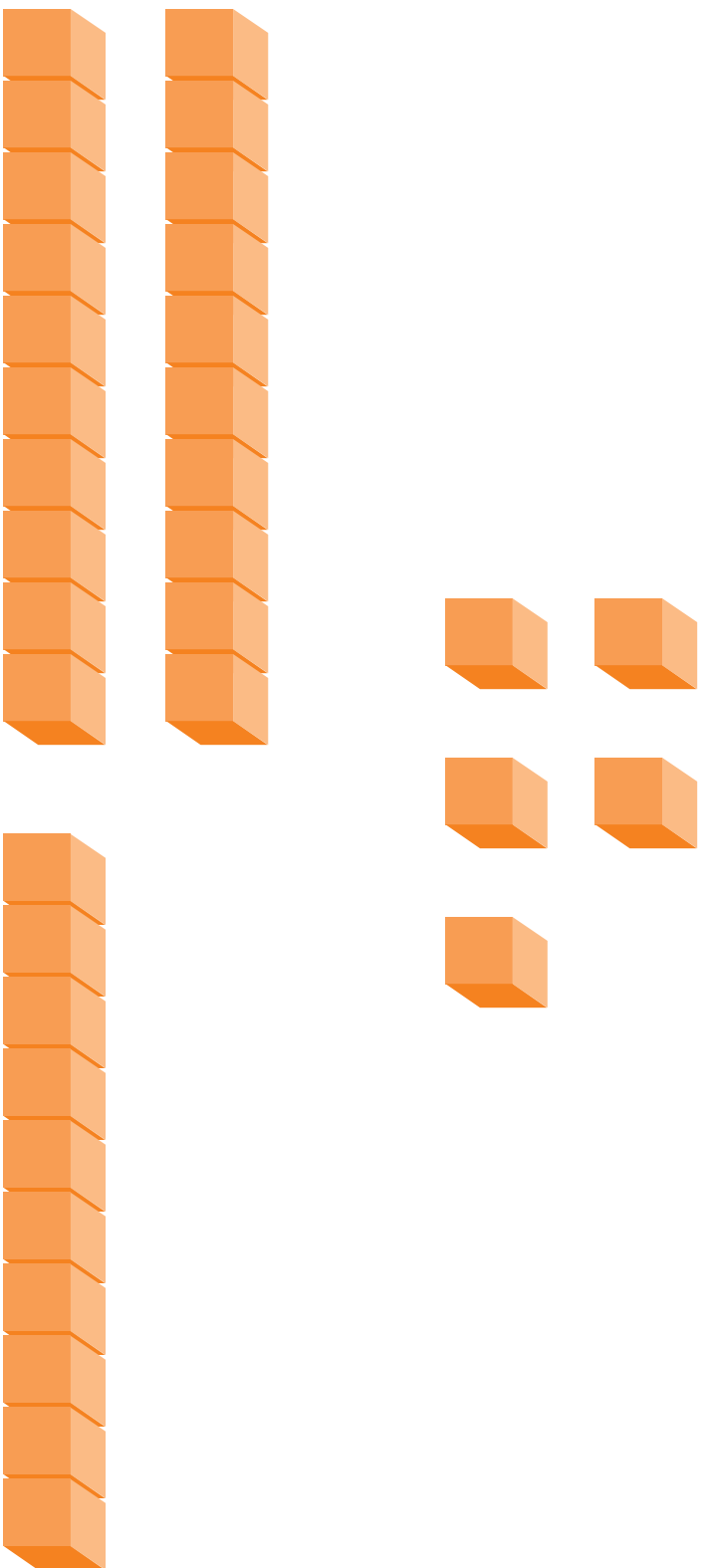
Materials available



Correct response

Using base-ten blocks: 1 long and 4 units
Using linking cubes: 10 cubes connected and 4 loose cubes

What's the Number?



Item D107

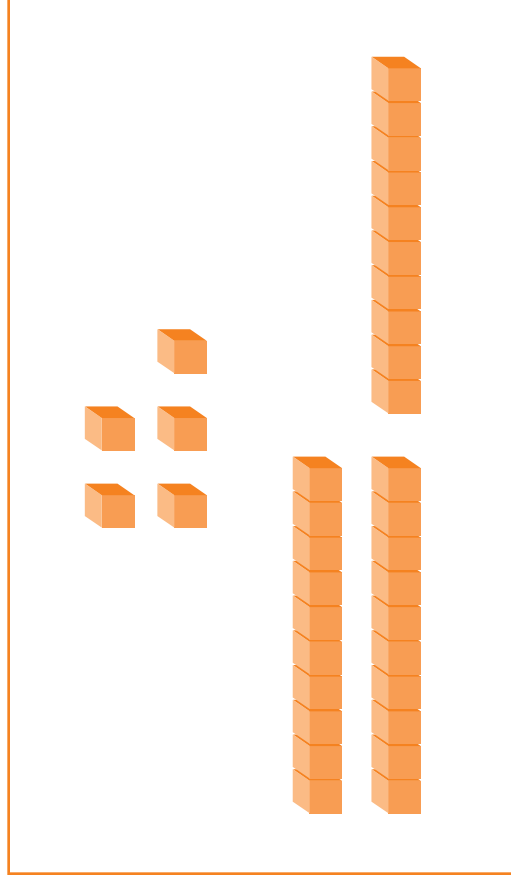
Place Value

7.2B

Translates among place value models, count words, numerals (2-digit numbers)

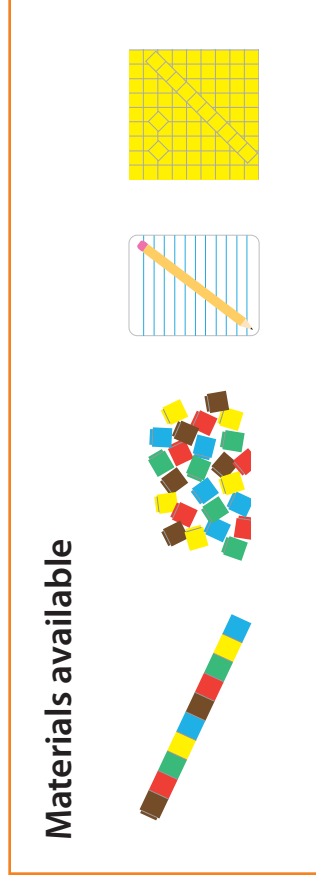
What's the Number?

- Place base-ten blocks, linking cubes (sticks of tens and loose cubes) within easy reach.
- Say, “**This picture shows a collection of base-ten blocks. What number is shown by the blocks in this picture?**”



For further diagnosis

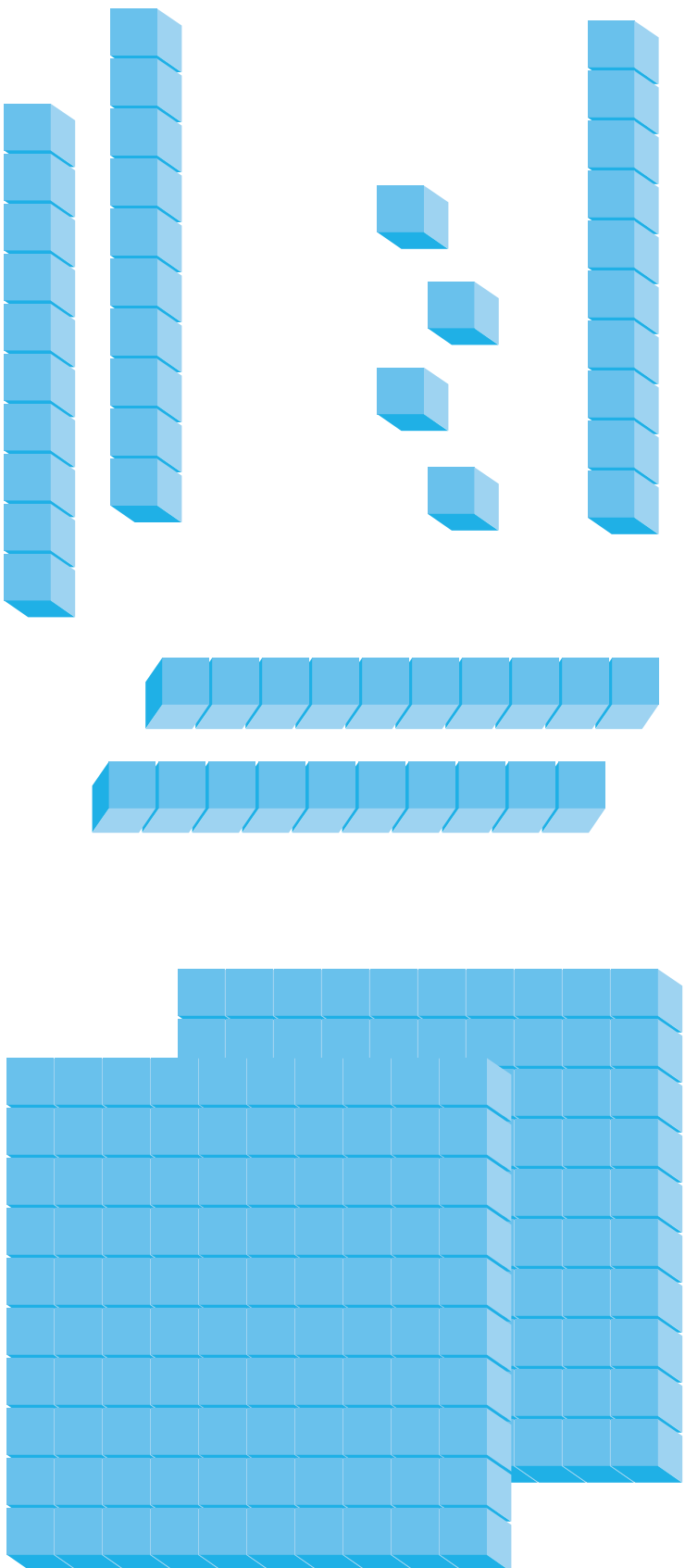
- Does the student count by ones or does he or she count by tens and then by ones?
- Does the student use physical base-ten blocks to solve the problem?
- Does the student recognize each long stick as a unit of 10?



Correct response

Uses tens and ones to get 35

What's the Number?



Item D108

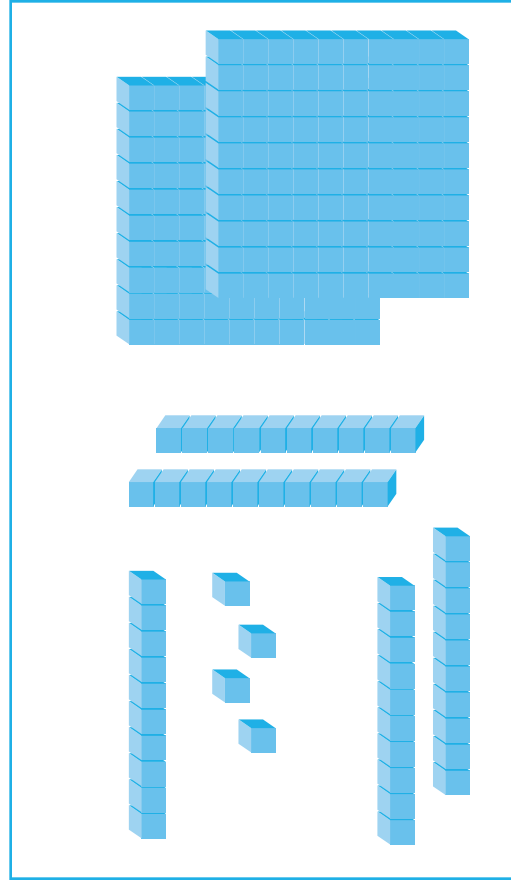
Place Value

7.2.C

Translates among place value models, count words, numerals (3-digit numbers)

What's the Number?

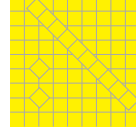
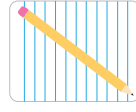
- Say, “This picture shows a collection of base-ten blocks like the ones you have. What is the number represented by the blocks in this picture?”



For further diagnosis

- Does the student count by hundreds, tens, and then by ones?
- Does the student use physical base-ten blocks to solve the problem?
- Does the student recognize each long stick as a unit of 10? Each flat block as a unit of 100?
- Ask the student to write the number “two hundred fifty-four.” Does the student write 200504?
- If the student writes 254, ask him to identify how the blocks in the picture show each digit.

Materials available



Correct response

254

Write the Number

5 ones and 1 ten

Write the Number

- Hand the student paper and a writing tool.
- Say, “Write the number that is made of 5 ones and 1 ten.”

5 ones and 1 ten

Materials available



For further diagnosis

- Does the student write 51?
- Ask the student to take out base-ten blocks to represent the value.
- Ask the student to build 15 using base-ten blocks.

Correct response

Writes “15”

Write the Number

8 ones and 4 tens

Write the Number

- Say, “Write the number that is made of 8 ones and 4 tens.”

8 ones and 4 tens

Materials available



For further diagnosis

- Does the student write 84?
- Ask the student to take out base-ten blocks to represent the value.
- Ask the student to build 48 using base-ten blocks.

Correct response

Writes “48”

Write the Number

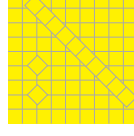
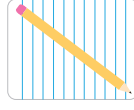
7 tens and 4 ones and 5 hundreds

Write the Number

- Say, “Write the number that is made of 7 tens and 4 ones and 5 hundreds.”

7 tens and 4 ones and 5 hundreds

Materials available



Correct response

Writes “574”

For further diagnosis

- Does the student write 745?
- Ask the student to take out base-ten blocks to represent the value.
- Ask the student to build 574 using base-ten blocks.

Sticker Collection

We have 23 stickers. I am putting them in my album with 10 stickers on each page.

How many full pages will I have?

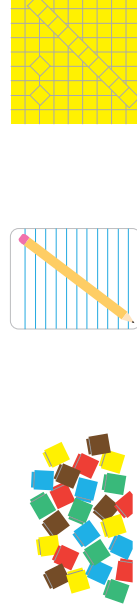
How many stickers will be left over?

Sticker Collection

- Read the problem aloud.

We have 23 stickers. I am putting them in my album with 10 stickers on each page.
How many full pages will I have?
How many stickers will be left over?

Materials available



For further diagnosis

- Does the student recognize that the digits in the numeral give the answer to the problem?
- Does the student draw a picture to solve the problem?
- Does the student use base-ten blocks or unit cubes to model the situation?

Correct response

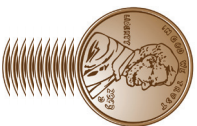
2 (full pages) with 3 (stickers left over)

Stacking Pennies

We have 78 pennies. We are putting them in stacks of 10 pennies each.

How many full stacks can we make?

How many pennies will be left over?



Stacking Pennies

- Read the problem aloud.

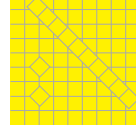
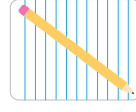
We have 78 pennies. We are putting them in stacks of 10 pennies each.

How many full stacks can we make?

How many pennies will be left over?



Materials available



For further diagnosis

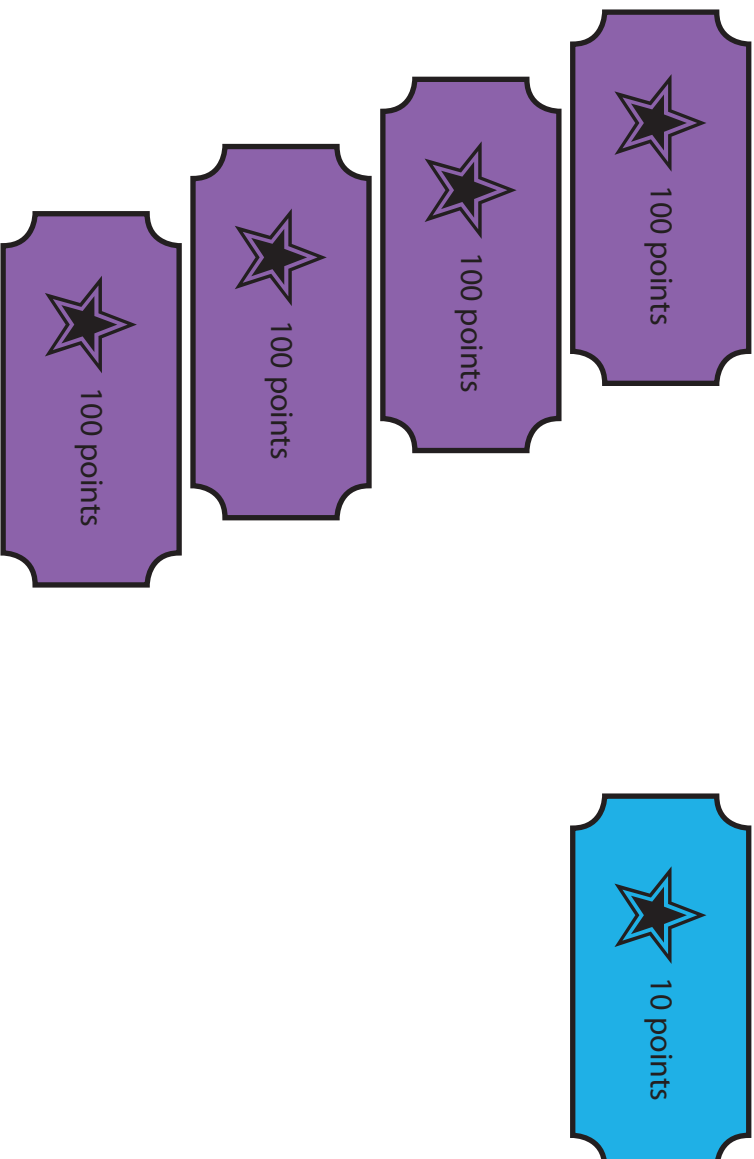
- Does the student recognize that the digits in the numeral give the answer to the problem?
- Does the student draw a picture to solve the problem?
- Does the student use base-ten blocks or unit cubes to model the situation?

Correct response

7 (full stacks) with 8 (pennies left over)

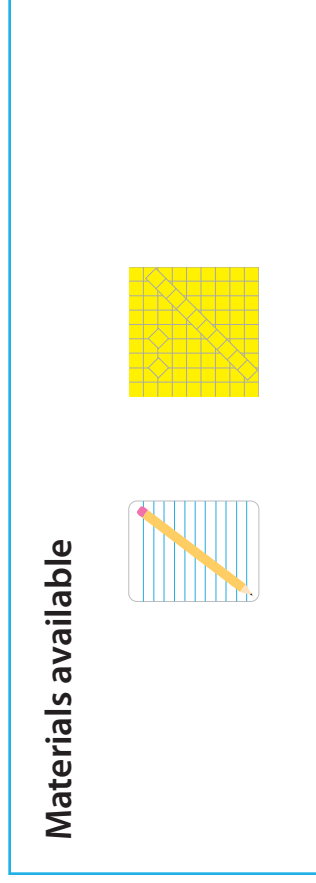
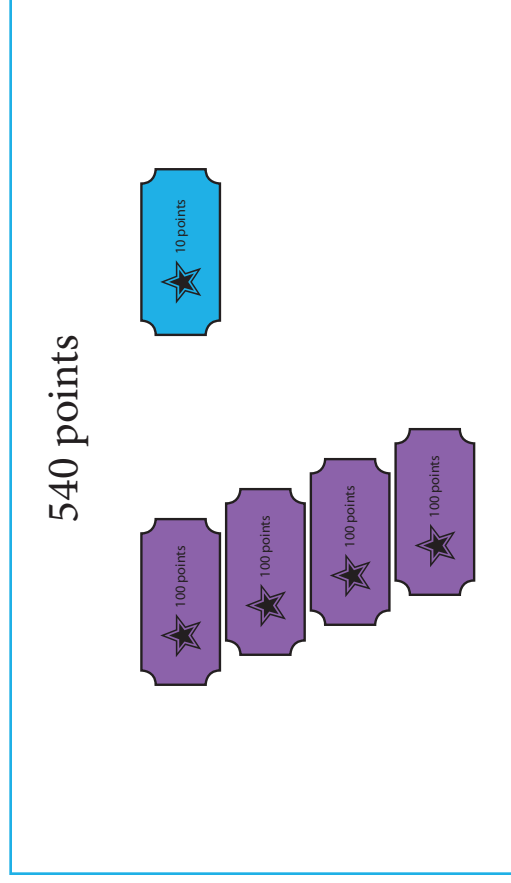
How Many More Tickets?

540 points



How Many More Tickets?

- Say, “How many more purple tickets and how many more blue tickets would you need to have a total of 540 points?”



For further diagnosis

- Does the student recognize that ticket values correspond to the place value of the digits?
- Does the student use base-ten blocks to represent the problem?
- Ask the student to find the value of the collection as shown.

Correct response

1 more purple ticket and 3 more blue tickets (or 13 more blue tickets)

How Many Books?

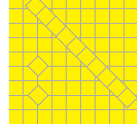
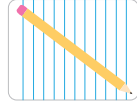
A store had 43 books about animals. They got 24 new animal books. Now how many animal books does the store have?

How Many Books?

- Read the problem aloud.

A store had 43 books about animals. They got 24 new animal books. Now how many animal books does the store have?

Materials available



For further diagnosis

- Ask the student to explain how he or she found the solution.
- Does the student use base-ten materials to represent the problem? Number line? Other model?
- Does the student use a standard algorithm or another process (e.g., partial sums or a left-to-right process)?

Correct response

Uses a strategy other than counting by ones and says “67”

How Many Children?

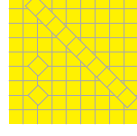
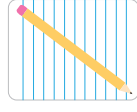
There were 37 children playing in the park.
Eighteen more children came to the park
to play. Now how many children are
playing in the park?

How Many Children?

- Read the problem aloud.

There were 37 children playing in the park.
Eighteen more children came to the park
to play. Now how many children are
playing in the park?

Materials available



For further diagnosis

- Ask the student to explain how he or she found the solution.
- Does the student use base-ten materials to represent the problem? Number line? Other model?
- Does the student use a standard algorithm or another process (e.g., partial sums or a left-to-right process)?
- How does the student account for the regrouping process?

Correct response

Uses a strategy other than counting by ones and says “55”

Explain the Process

635

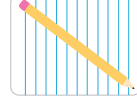
$$\begin{array}{r} 635 \\ + 296 \\ \hline \end{array}$$

Explain the Process

- Say, “Use your pencil and paper to find the answer. Explain what you did and why it works.”

$$\begin{array}{r} 635 \\ + 296 \\ \hline \end{array}$$

Materials available



For further diagnosis

- How does the student account for the regrouping process?
- Does the student use the standard algorithm or a different method (e.g., partial sums or a left-to-right process?)
- Ask the student to solve a different 3-digit problem that requires only one regrouping.
- Ask the student to solve a 2-digit problem that involves regrouping.
- Ask the student to explain the method in terms of place value.

Correct response

931

How Many Cars?

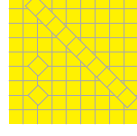
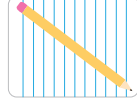
There were 88 cars in a parking lot.
Fourteen cars drove away.
Now how many cars are in the parking lot?

How Many Cars?

- Read the problem aloud.

There were 88 cars in a parking lot.
Fourteen cars drove away.
Now how many cars are in the parking lot?

Materials available



For further diagnosis

- Ask the student to explain how he or she found the solution.
- Does the student use base-ten materials to represent the problem? Number line? Other model?
- Does the student use a standard algorithm or another process (e.g., partial differences or a left-to-right process)?

Correct response

Uses a strategy other than counting by ones and says “74”

How Many Wristbands?

For the school carnival my class made 71 wristbands to sell. We sold 35 of them on the first day. How many wristbands do we have now?

How Many Wristbands?

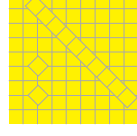
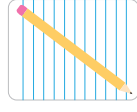
- Read the problem aloud.

For the school carnival my class made 71 wristbands to sell. We sold 35 of them on the first day. How many wristbands do we have now?

For further diagnosis

- Ask the student to explain how he or she found the solution.
- Does the student use base-ten materials to represent the problem? Number line? Other model?
- Does the student use a standard algorithm or another process (e.g., partial differences or a left-to-right process)?
- How does the student account for the regrouping process?

Materials available



Correct response

Uses a strategy other than counting by ones and says “36”

Explain the Process

472

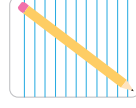
- 187

Explain the Process

- Say, “Use your pencil and paper to find the answer. Explain to me what you did and why it works.”

$$\begin{array}{r} 472 \\ -187 \\ \hline \end{array}$$

Materials available



For further diagnosis

- How does the student account for the regrouping process?
- Does the student use the standard algorithm or a different method (e.g., partial differences or a left-to-right process?)
- Ask the student to solve a different 3-digit problem that requires only one regrouping.
- Ask the student to solve a 2-digit problem that involves regrouping.
- Ask the student to explain the method in terms of place value.

Correct response

285

End of Concept Area 7

Concept Area 8

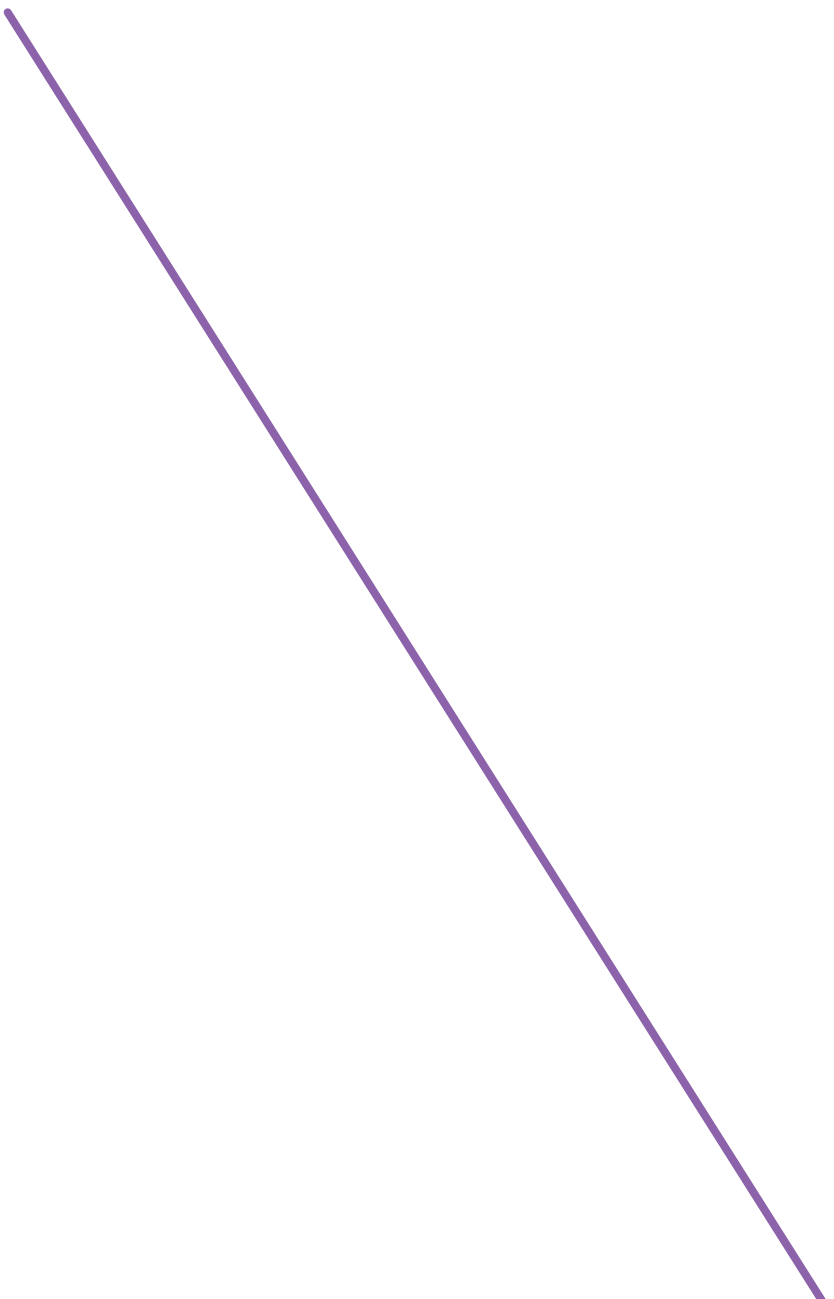
Competencies for Measurement

Does the student understand and use these big ideas?

- Attributes of objects can be measured or compared
- Uniform units allow attributes of objects to be quantified
- Measures can be determined by repeating a unit or using a tool

Learning goal	Performance level		
	A	B	C
8.1 Makes comparisons based on measurable attributes	Compares length directly	Compares length indirectly	<i>Orders 3 objects by length</i>
8.2 Measures length	By laying multiple length units end-to-end	By iterating a single length unit	<i>By using a ruler</i>
8.3 Understands units		<i>Recognizes the need for equal-sized units</i>	<i>Estimates change in measurement based on change in unit</i>

Which Is Longer?



Item D121

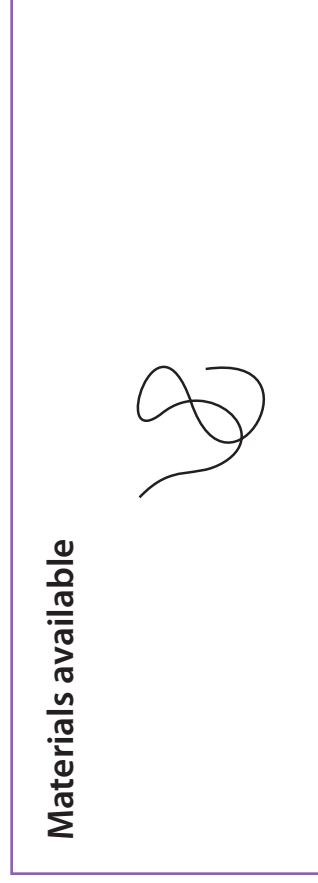
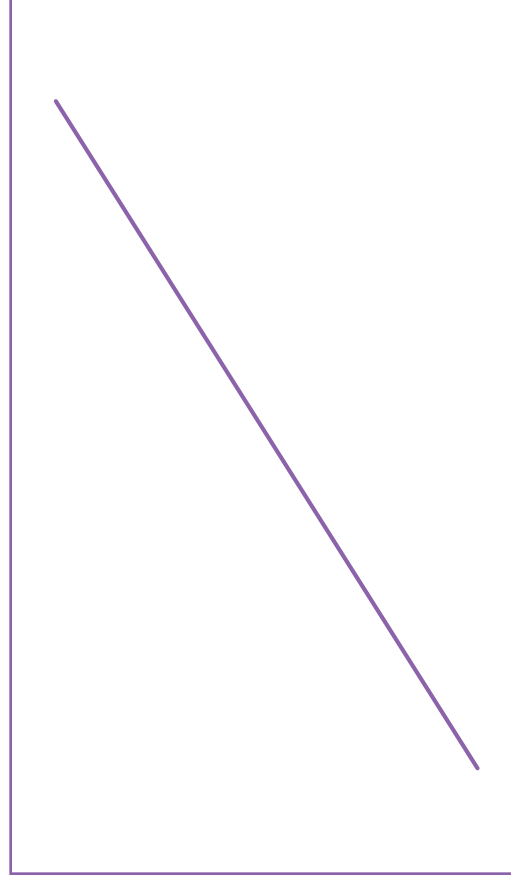
Measurement

8.1A

Makes comparisons based on measurable attributes (compares length directly)

Which Is Longer?

- Ball or coil the precut string (length: 9 inches). Hand it to the student.
- Say, **“Which is longer, this string or the line on this page?”**



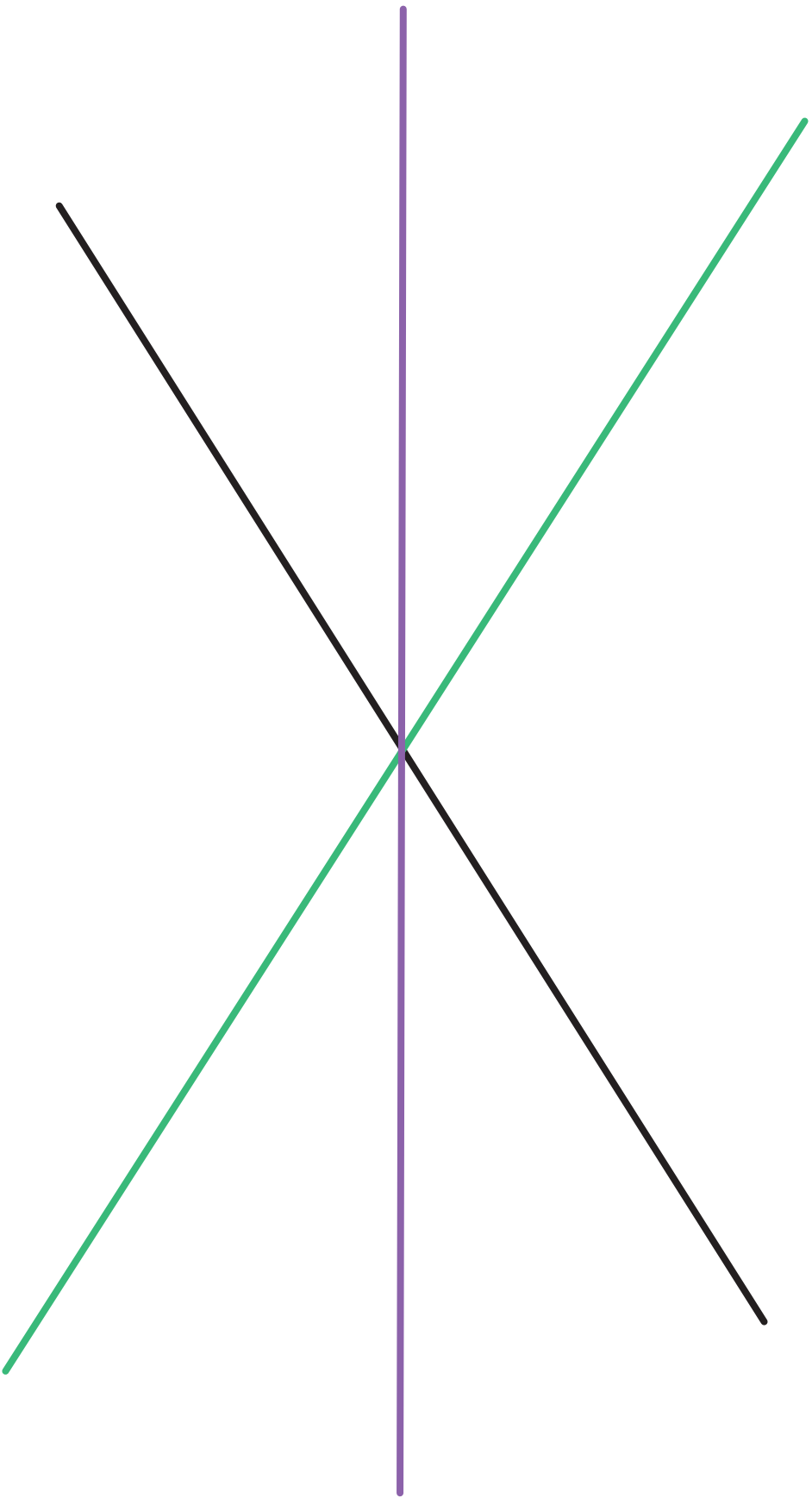
Correct response

String

For further diagnosis

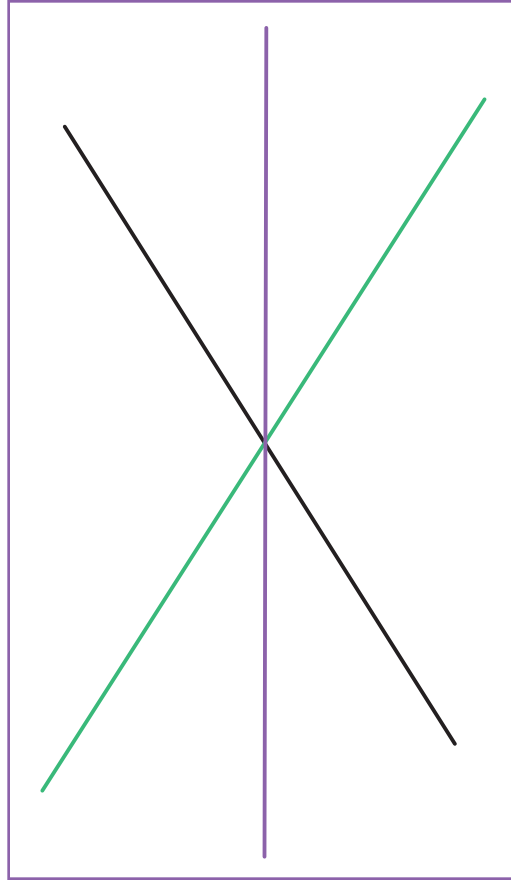
- Does the student stretch the string to its full length?
- Does the student align the end of the string with the end of the segment?

Which Are the Same Length?



Which Are the Same Length?

- Say, “There are three lines on this page. Which lines are the same length?”
- Say, “You may use the string, linking cubes, or another tool to help you.”



Materials available



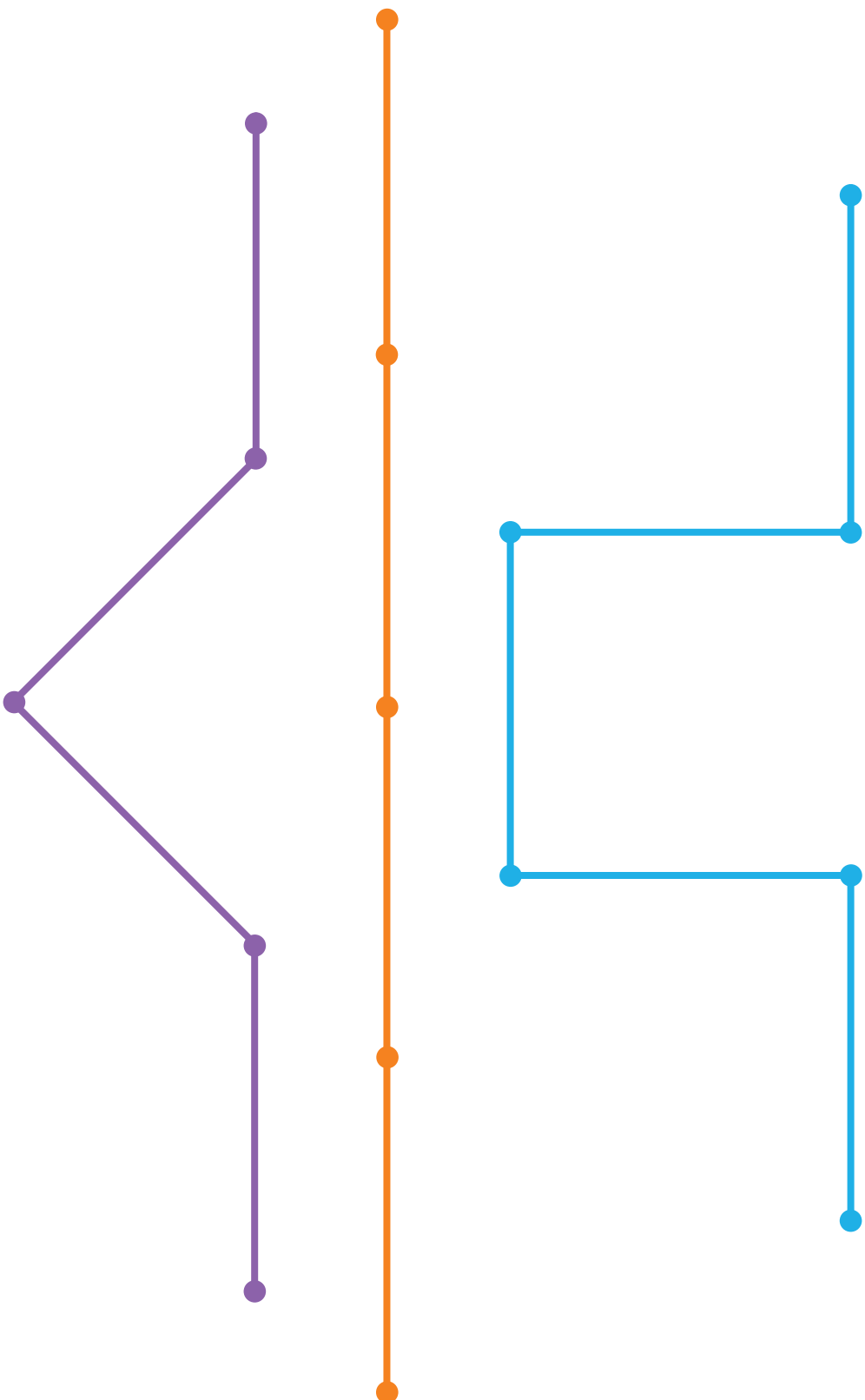
For further diagnosis

- Does the student use a tool to make the comparison (pencil, string, linking cubes)?
- Does the student exclusively consider the position of the endpoints rather than the distance between endpoints when making the judgment?

Correct response

Green and purple lines

Which Path?



Item D123

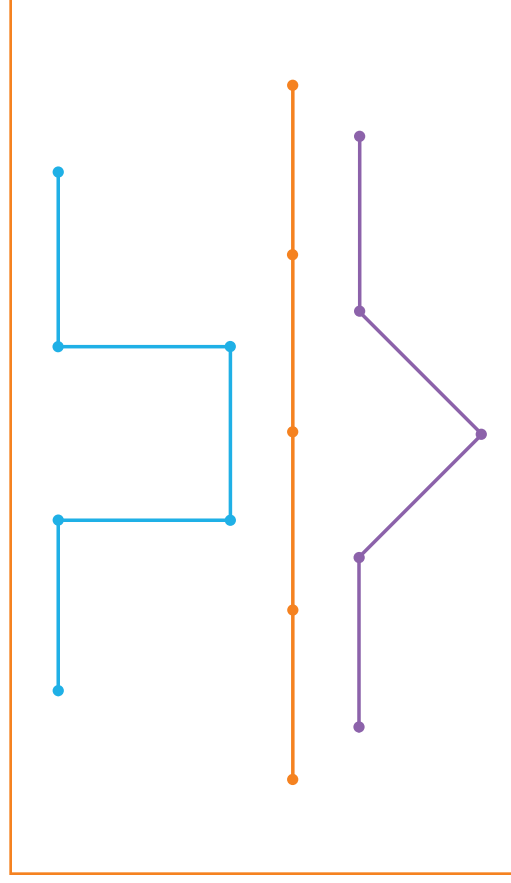
Measurement

8.1C

Makes comparisons based on measurable attributes (orders 3 objects by length)

Which Path?

- Say, “Look at the three paths. Pretend a bug took a walk along each of these paths. Which is the longest path (longest walk)?”



Materials available



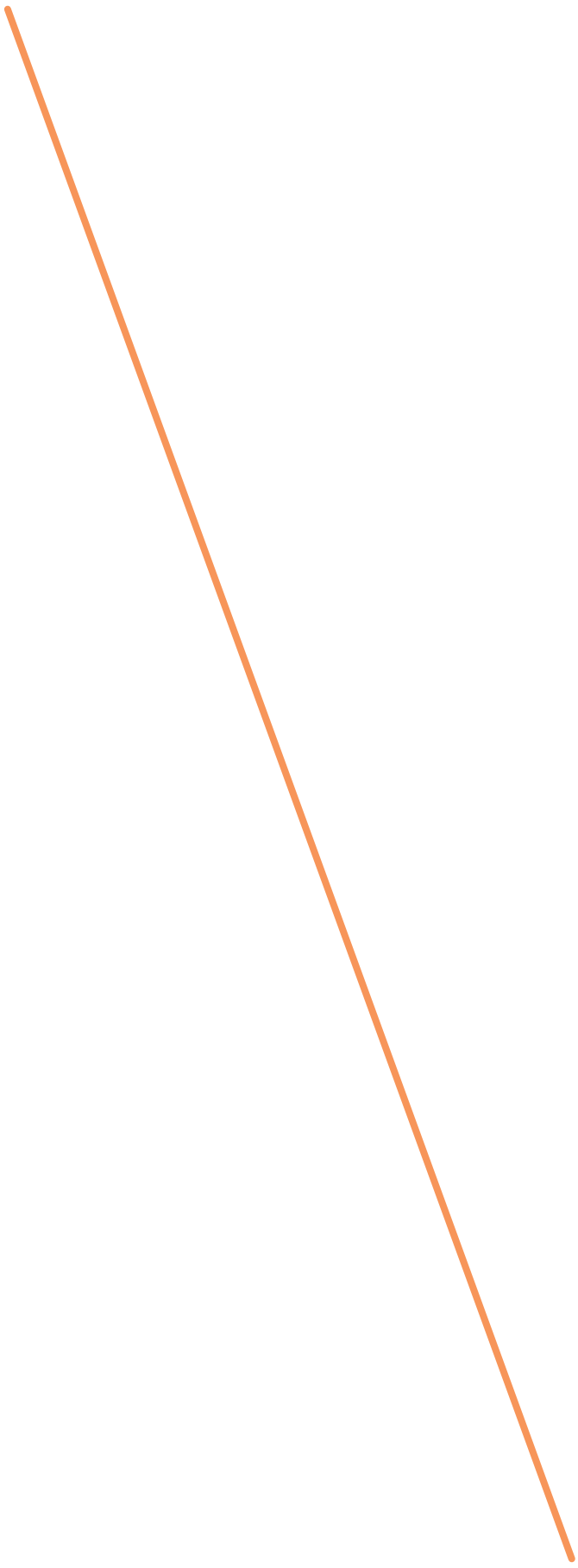
Correct response

Blue (top is longest)

For further diagnosis

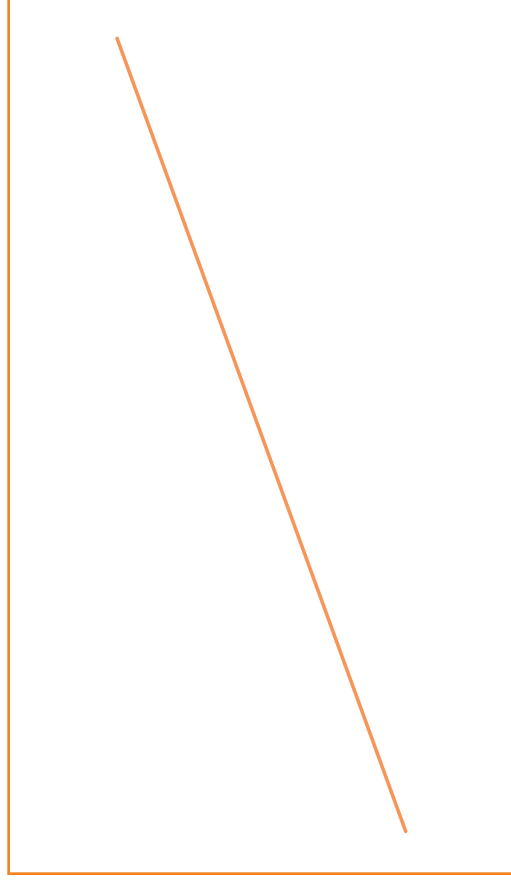
- Does the student recognize that each segment is the same length?
- Does the student exclusively consider endpoints when making the judgment, rather than length?
- Ask the student to use a tool (linking cubes, ruler, pencil, fingers) to make the decision.

Measure the Line



Measure the Line

- Hand the student a supply of small paper clips.
- Say, “Measure the length of this line using these paper clips.”
- Say, “How long is the line in paper clips?”



Materials available



Correct response

Approximately 8 (depends on size of paper clip)

For further diagnosis

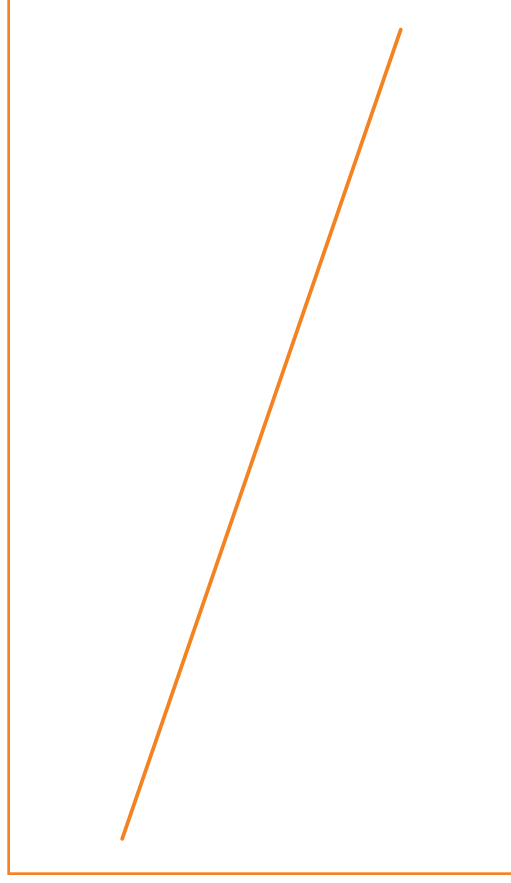
- Does the student accurately line up paper clips end to end with no gaps or overlaps?
- Does the student line up the paper clips with the endpoints of the line?

How Long Is the Line?



How Long Is the Line?

- Hand the student one large paper clip.
- Say, “Measure the length of this line using this paper clip.”
- Say, “How long is the line in paper clips?”



Materials available



one large

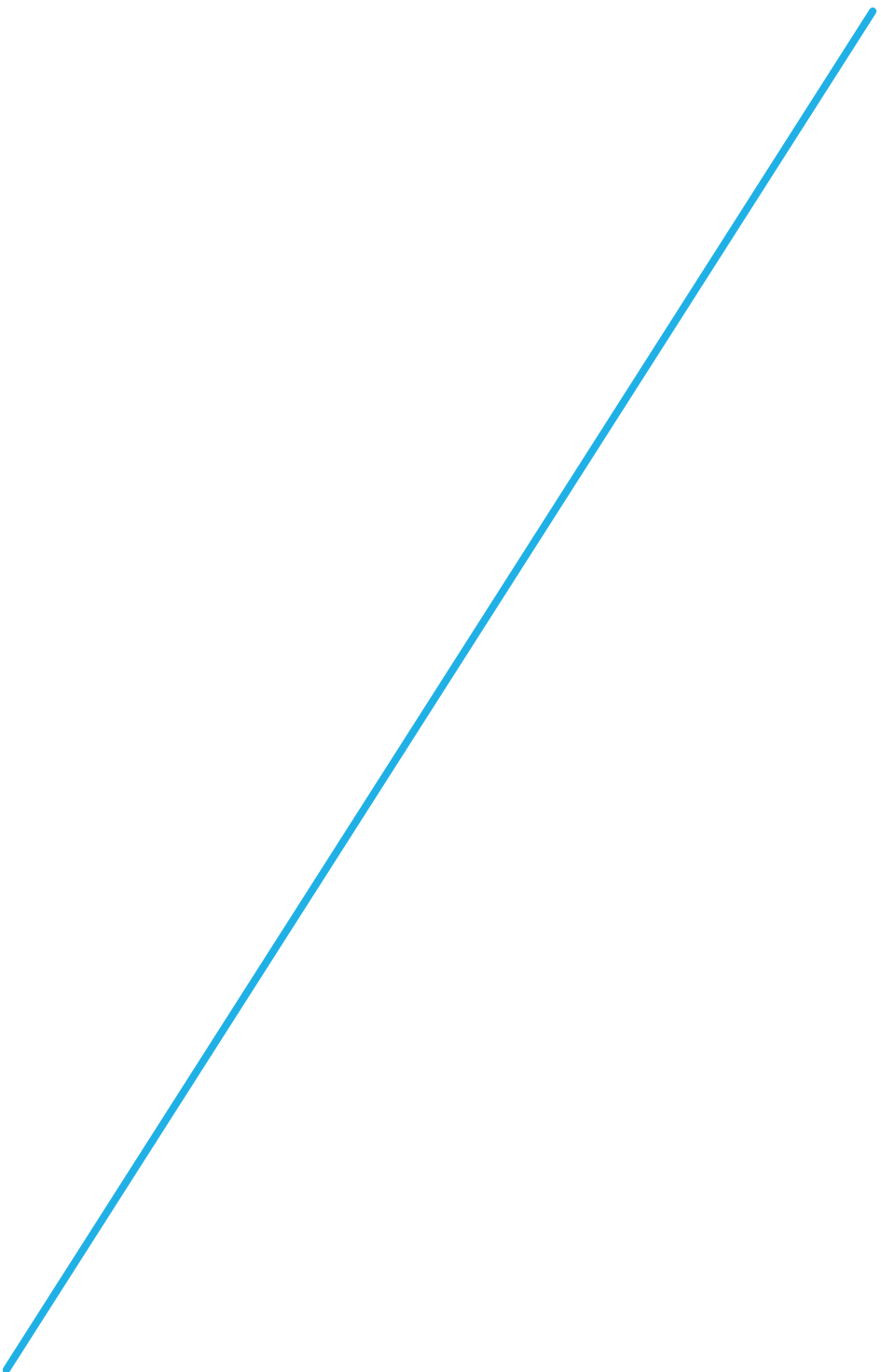
Correct response

Approximately 5 (depends on the size of the paper clip)

For further diagnosis

- Does the student keep track of the successive placement of the paper clip?
- Does the student accurately place the paper clip reasonably to account for the end-to-end placement?
- Draw another line on a sheet of paper. Have the student mark with pencil the end of the paper clip after each placement.

How Long Is the Line?

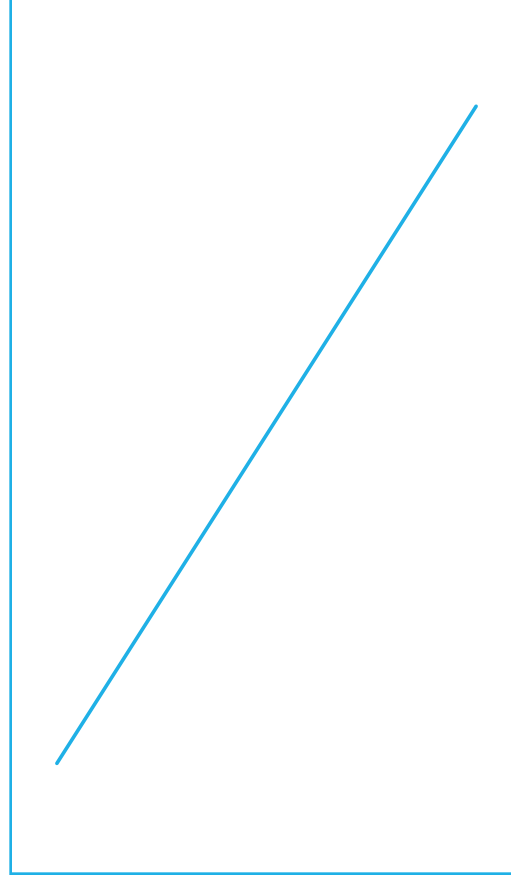


How Long Is the Line?

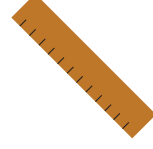
- Hand the student a ruler.
- Say, “Measure the length of this line using this ruler.”
- Say, “How long is the line in inches?”
- If the student is using a ruler with both inches and centimeters, direct the student to use the side showing inches.

For further diagnosis

- Does the student have familiarity with the ruler as a tool for measuring?
- Ask the student what this tool is used for.
- Does the student accurately line up the 0 point of the ruler with the endpoint of the line?



Materials available



Correct response

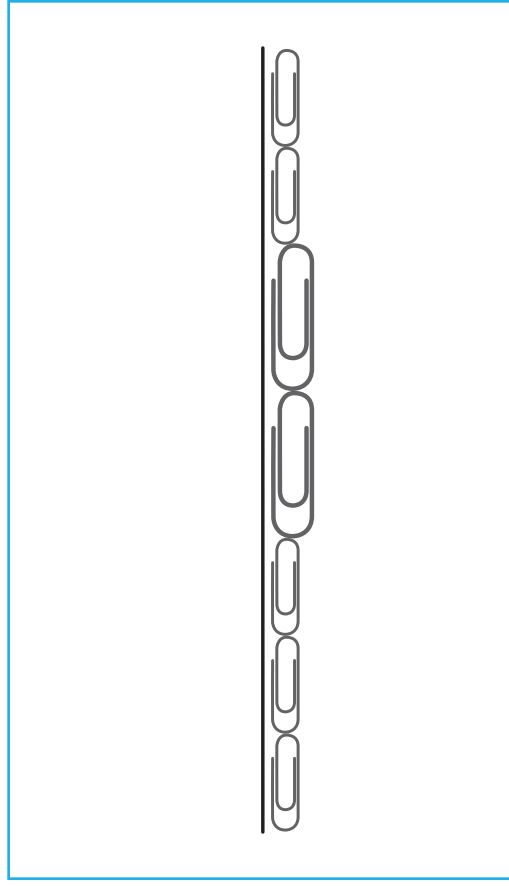
9 (inches)

Is This Correct?



Is This Correct?

- Say, “A student placed the paper clips on the line like this and said that the length of the line is seven paper clips. Is this correct? Why or why not?”



Materials available

none

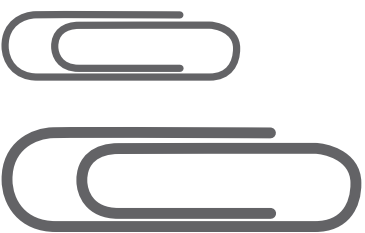
Correct response

No, because the paper clips are not all the same size

For further diagnosis

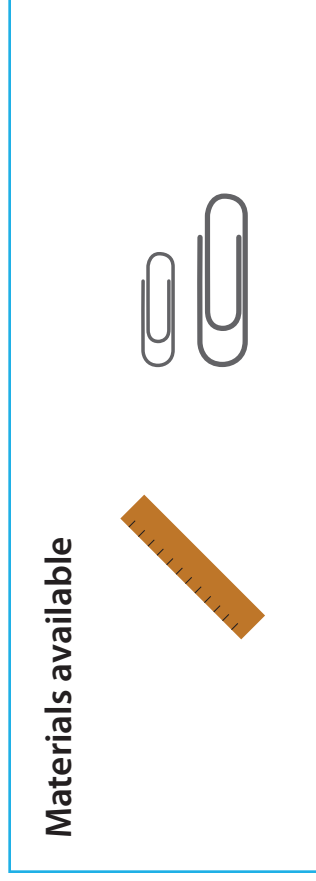
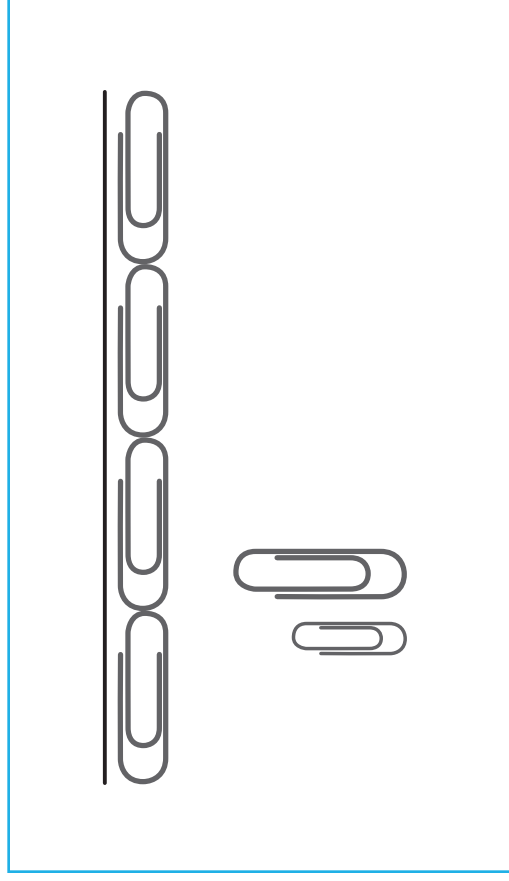
- Does the student recognize the need for same-sized paper clips?
- Hand the student a supply of uniform paper clips and ask him to measure the line. How does he or she reconcile the difference in measurements?

How Many Small Paper Clips?



How Many Small Paper Clips?

- Say, “If you measured this line using the small paper clips shown in the picture, about how many of the small paper clips would it take?”



Correct response

Anywhere between 5 and 7 paper clips

For further diagnosis

- Does the student give a value greater than four?
- Does the student use a tool (fingers, cubes, paper clips) to come to an answer?
- Ask the student how the number of paces would compare if a child paced off the width of the room and if an adult paced off the width of the room.

End of Content Area 8

Concept Area 9



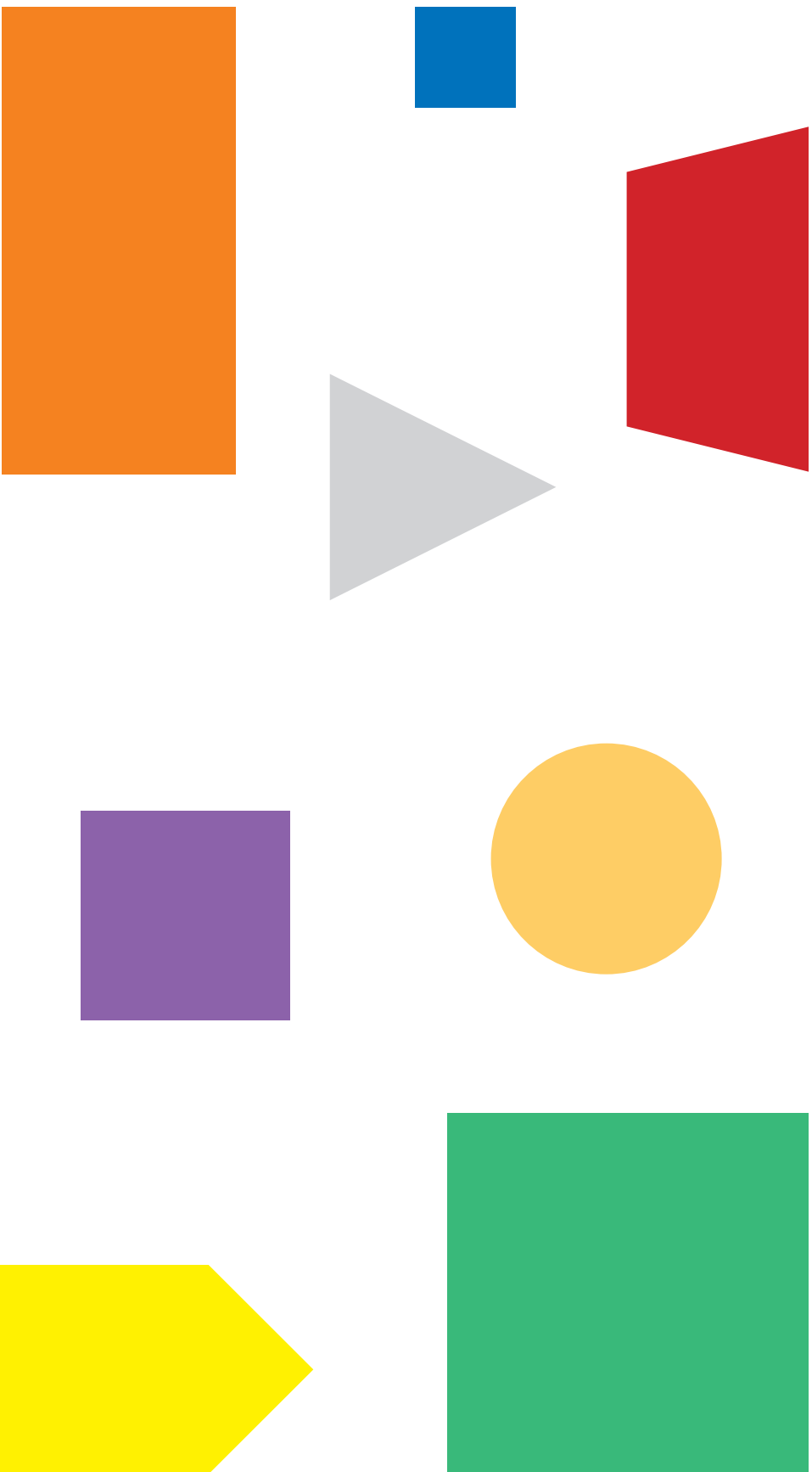
Competencies for Geometry

Does the student understand and use these big ideas?

- Geometric shapes can be identified by their attributes, regardless of their orientation
- Geometric shapes can be composed and decomposed into other shapes

Learning goal	Performance level		
	A	B	C
9.1 Identifies quadrilaterals in standard orientation	Squares	<i>Normal proportion rectangles</i>	<i>Rectangles with exaggerated aspect ratio</i>
9.2 Identifies triangles in standard orientation	<i>Equilateral triangles in point-up orientation</i>	<i>Nonequilateral triangles in point-up orientation</i>	
9.3 Identifies geometric figures in nonstandard orientation	<i>Squares</i>	Rectangles	Triangles
9.4 Identifies components and properties of shapes			<i>Identifies shapes based on their properties</i>
9.5 Composes geometric figures	Simple frames with distinct outlines	No frame provided	By substituting a combination of smaller shapes for a larger shape

Point to the Squares



Item D129

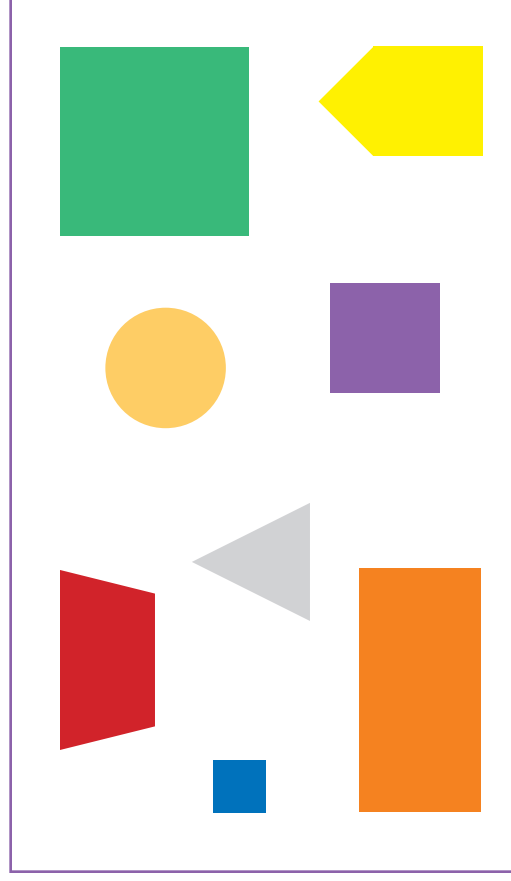
Geometry

9.1A

Identifies quadrilaterals in standard orientation (squares)

Point to the Squares

- Say, “Show me a square.”
- Say, “Can you find another square?”
- Repeat the question until the student says there are no more squares.



Materials available

none

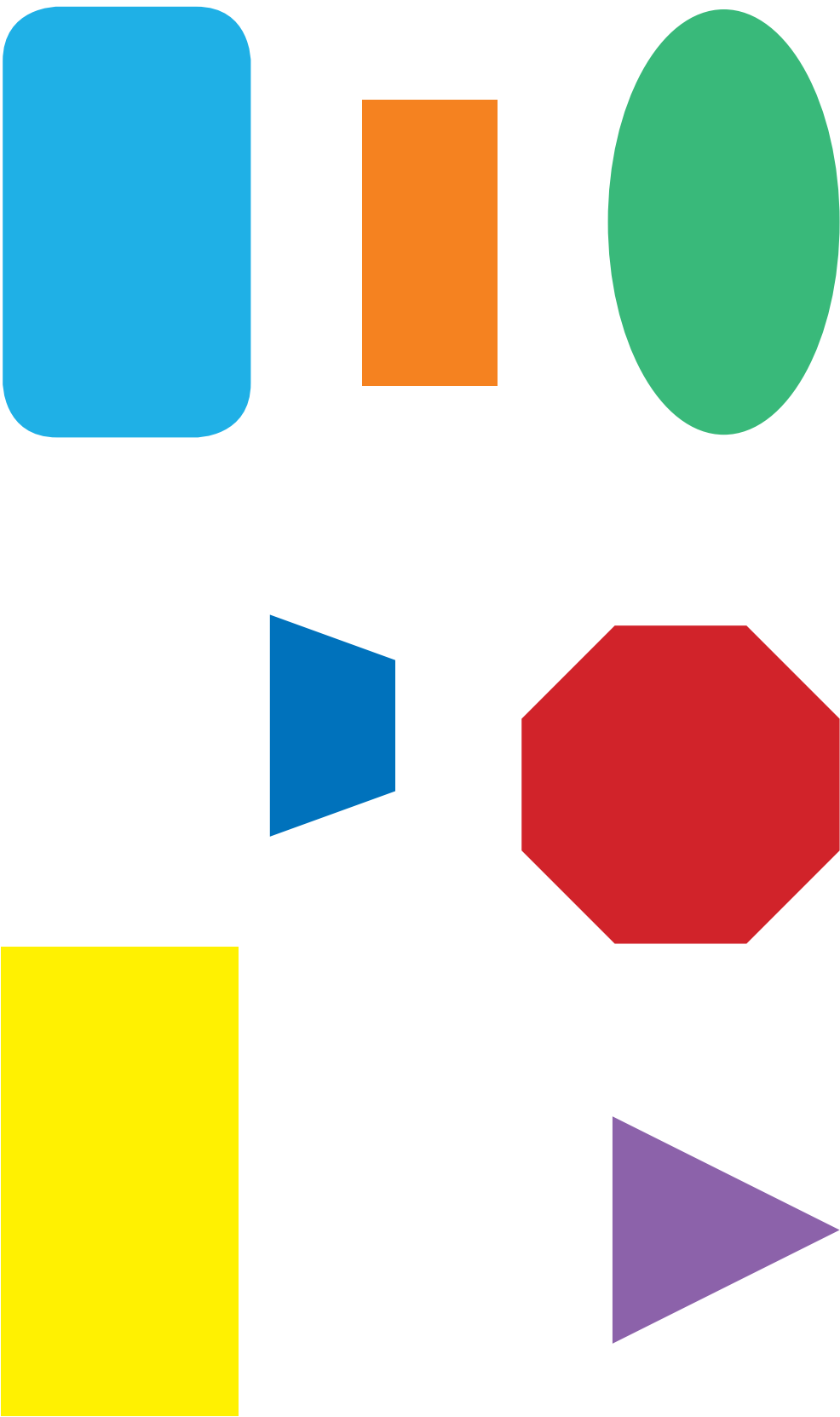
For further diagnosis

- Does the student accept all 4-sided figures as squares?
- Ask the student to draw a square.
- Ask the student what makes a square a square. Does he or she identify both characteristics (four equal sides, and four right angles or square corners)?

Correct response

Blue, green, and purple squares

Point to the Rectangles



Item D130

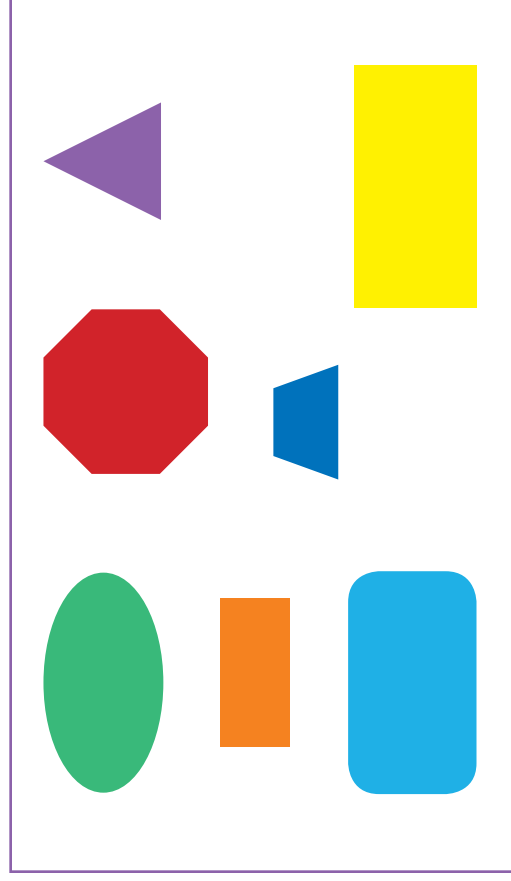
Geometry

9.1B

Identifies quadrilaterals in standard orientation (normal proportion rectangles)

Point to the Rectangles

- Say, “Show me a rectangle.”
- Say, “Can you find another rectangle?”
- Repeat the question until the student says there are no more rectangles.



Materials available

none

For further diagnosis

- Does the student accept the figure with rounded corners?
- Ask the student to draw a rectangle.
- Ask the student what makes a rectangle a rectangle. Does he or she identify both characteristics (four sides, and four right angles or square corners)?

Correct response

Orange and yellow rectangles

Point to the Rectangles



Item D131

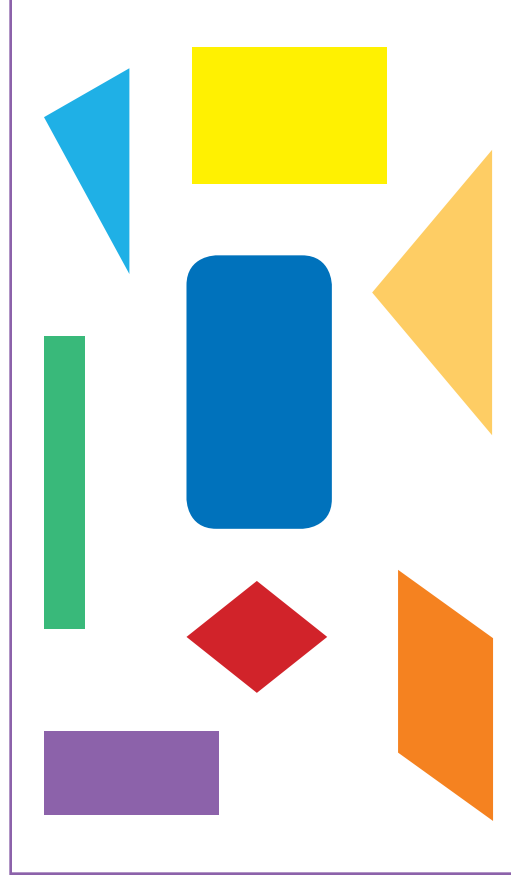
Geometry

9.1C

Identifies quadrilaterals in standard orientation (rectangles with exaggerated aspect ratio)

Point to the Rectangles

- Say, “Show me a rectangle.”
- Say, “Can you find another rectangle?”
- Repeat the question until the student says there are no more rectangles.



Materials available

none

For further diagnosis

- Does the student recognize rectangles that are long and skinny or short and fat?
- Does the student accept the figure with rounded corners?
- Ask the student to draw a rectangle.
- Ask the student what makes a rectangle a rectangle. Does he or she identify both characteristics (four sides, and four right angles or square corners)?

Correct response

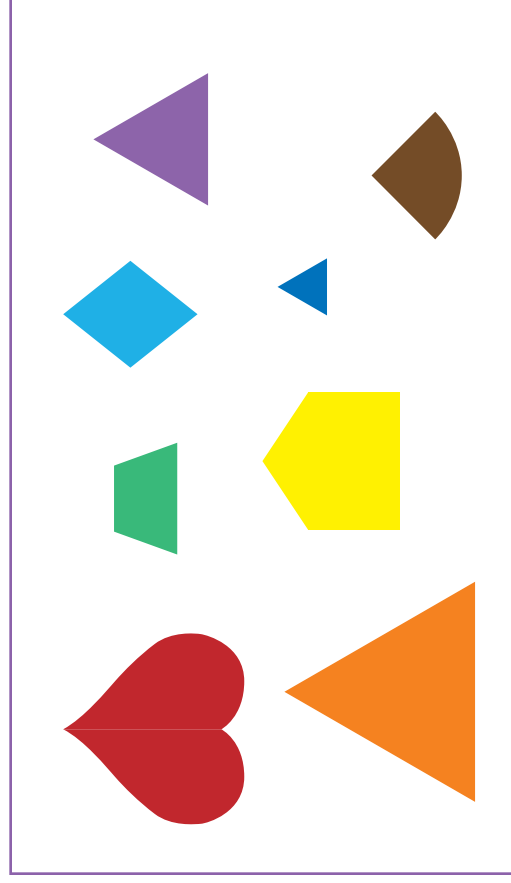
Purple, green, and yellow rectangles

Point to the Triangles



Point to the Triangles

- Say, “Show me a triangle.”
- Say, “Can you find another triangle?”
- Repeat the question until the student says there are no more triangles.



For further diagnosis

- Does the student identify all “point-up” shapes as triangles?
- Ask the student to draw as many different triangles as he or she can. Does he or she draw triangles that are non-equilateral?
- Ask the student what makes a triangle a triangle.
- Does he or she identify the characteristic of three sides?
- Does the student identify “pointy” as a characteristic of triangles?

Materials available

none

Correct response

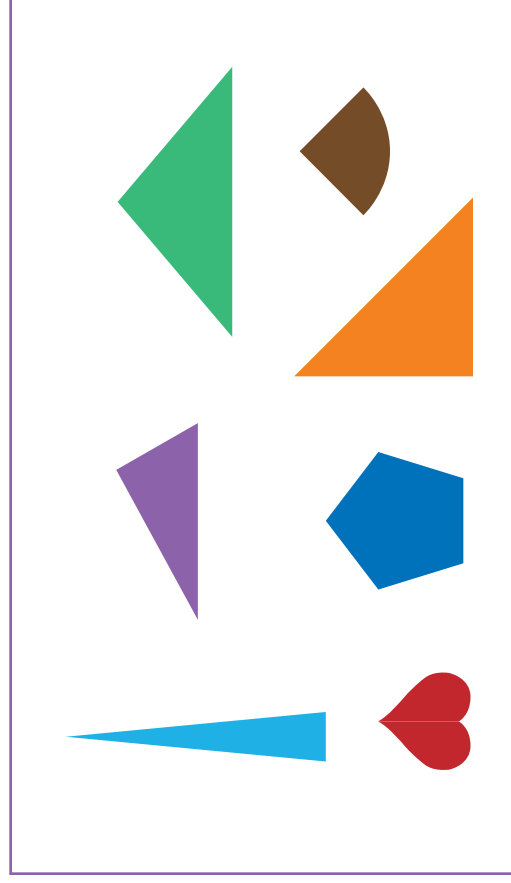
Orange, dark blue, and purple triangles

Point to the Triangles



Point to the Triangles

- Say, “Show me a triangle.”
- Say, “Can you find another triangle?”
- Repeat the question until the student says there are no more triangles.



For further diagnosis

- Does the student recognize triangles that are not equilateral?
- Does the student identify all “point-up” shapes as triangles?
- Ask the student to draw as many different triangles as he or she can. Does he or she draw triangles that are non-equilateral?
- Ask the student what makes a triangle a triangle.
- Does he or she identify the characteristic of three sides?

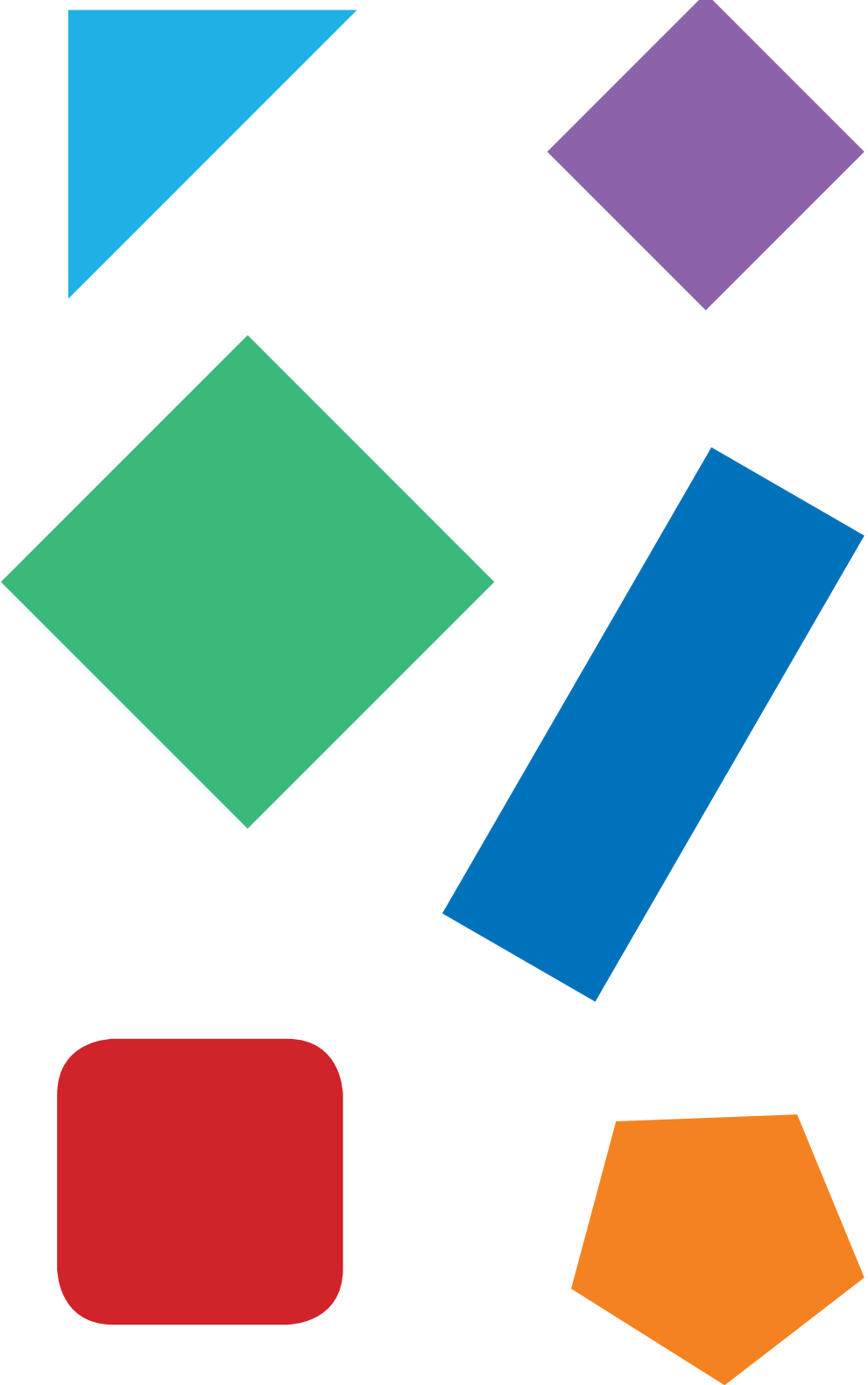
Materials available

none

Correct response

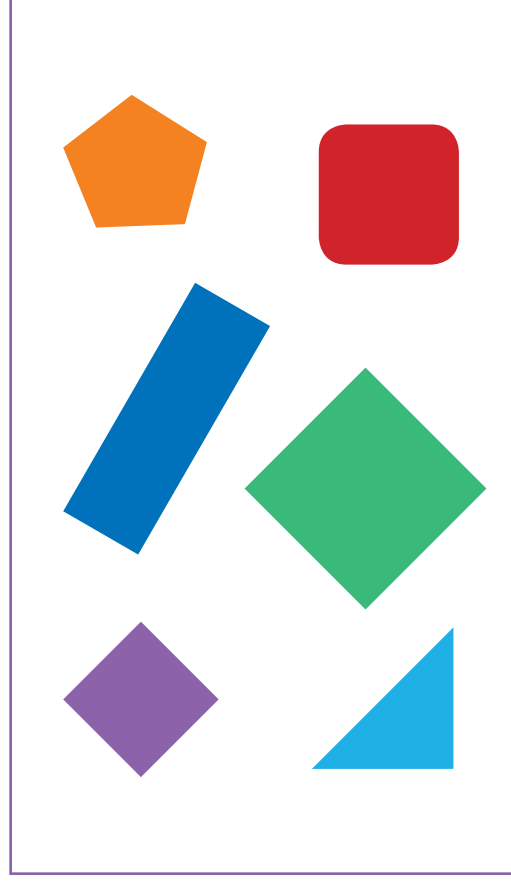
Blue, purple, green, and orange triangles

Point to the Squares



Point to the Squares

- Say, “Show me a square.”
- Say, “Can you find another square?”
- Repeat the question until the student says there are no more squares.



For further diagnosis

- Does the student recognize squares when they are in nonstandard orientation?
- Does the student accept the figure with rounded corners?
- Ask the student to name the purple and green shapes. Does he or she say “diamond?”
- Ask the student what makes a square a square. Does he or she identify both characteristics (four equal sides, and four right angles or square corners)?

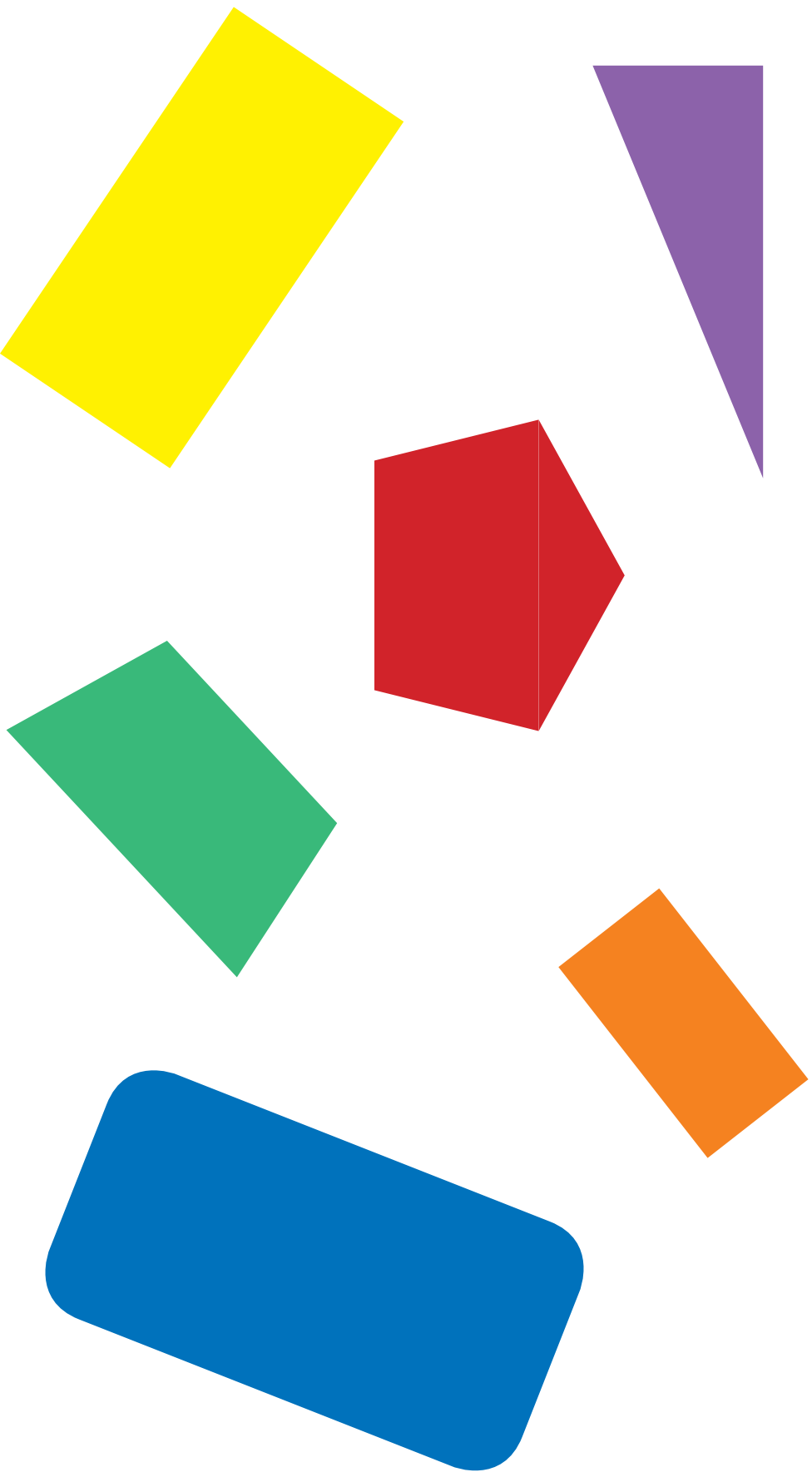
Materials available

none

Correct response

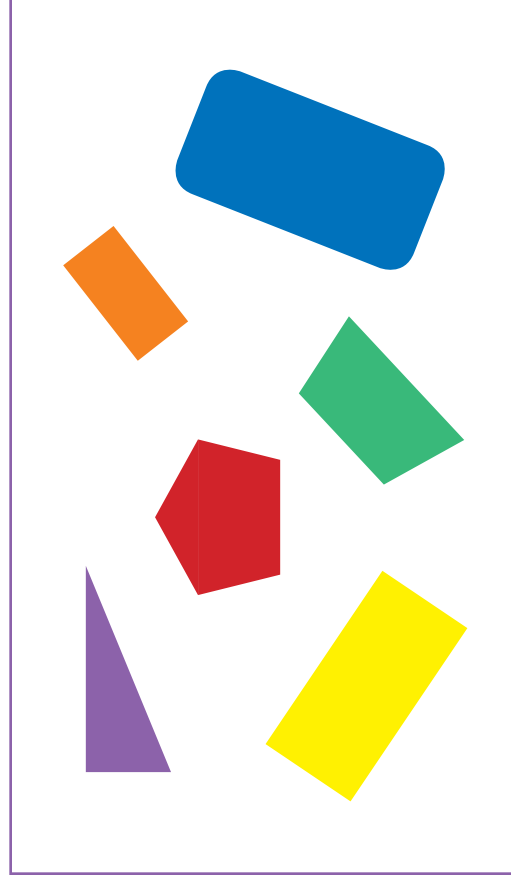
Purple and green squares

Point to the Rectangles



Point to the Rectangles

- Say, “Show me a rectangle.”
- Say, “Can you find another rectangle?”
- Repeat the question until the student says there are no more rectangles.



Materials available

none

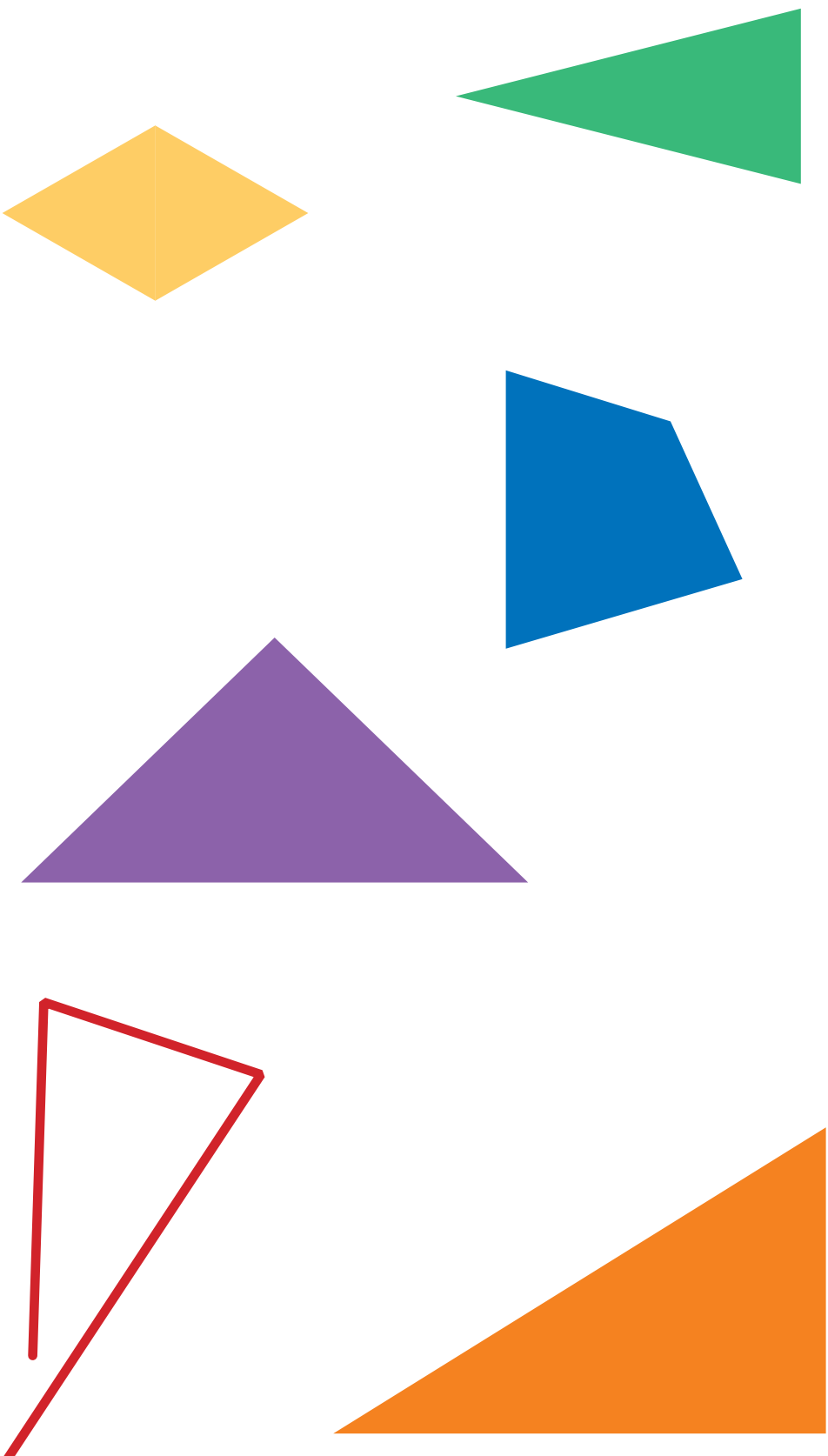
Correct response

Yellow and orange rectangles

For further diagnosis

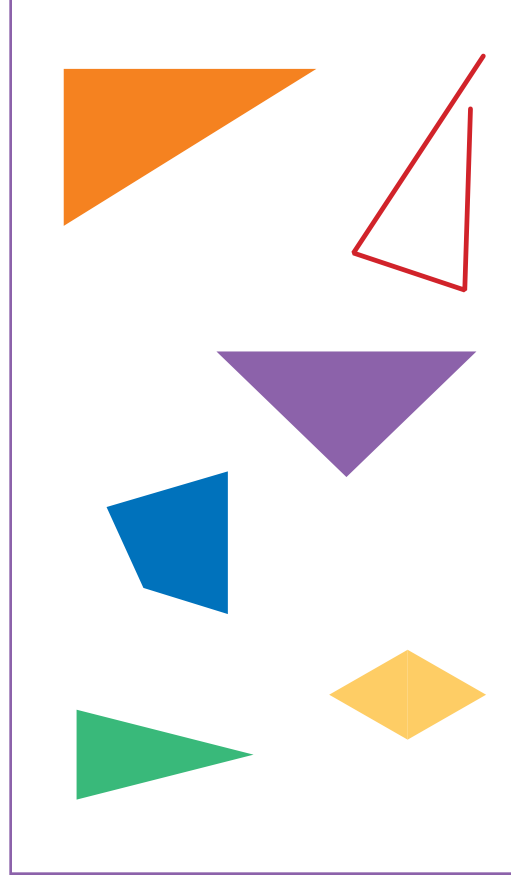
- Does the student recognize rectangles when they are in nonstandard orientation?
- Does the student accept the figure with rounded corners as a rectangle?
- Ask the student to draw a rectangle.
- Ask the student what makes a rectangle a rectangle. Does he or she identify both characteristics (four sides, and four right angles or square corners)?

Point to the Triangles



Point to the Triangles

- Say, “Show me a triangle.”
- Say, “Can you find another triangle?”
- Repeat the question until the student says there are no more triangles.



For further diagnosis

- Does the student recognize triangles when they are in nonstandard orientation?
- Does the student accept the nonclosed figure?
- Ask the student to draw as many different triangles as he or she can. Does he or she draw triangles that are non-equilateral?
- Ask the student what makes a triangle a triangle. Does he or she identify the characteristic of three sides and the fact that it must be a closed figure?

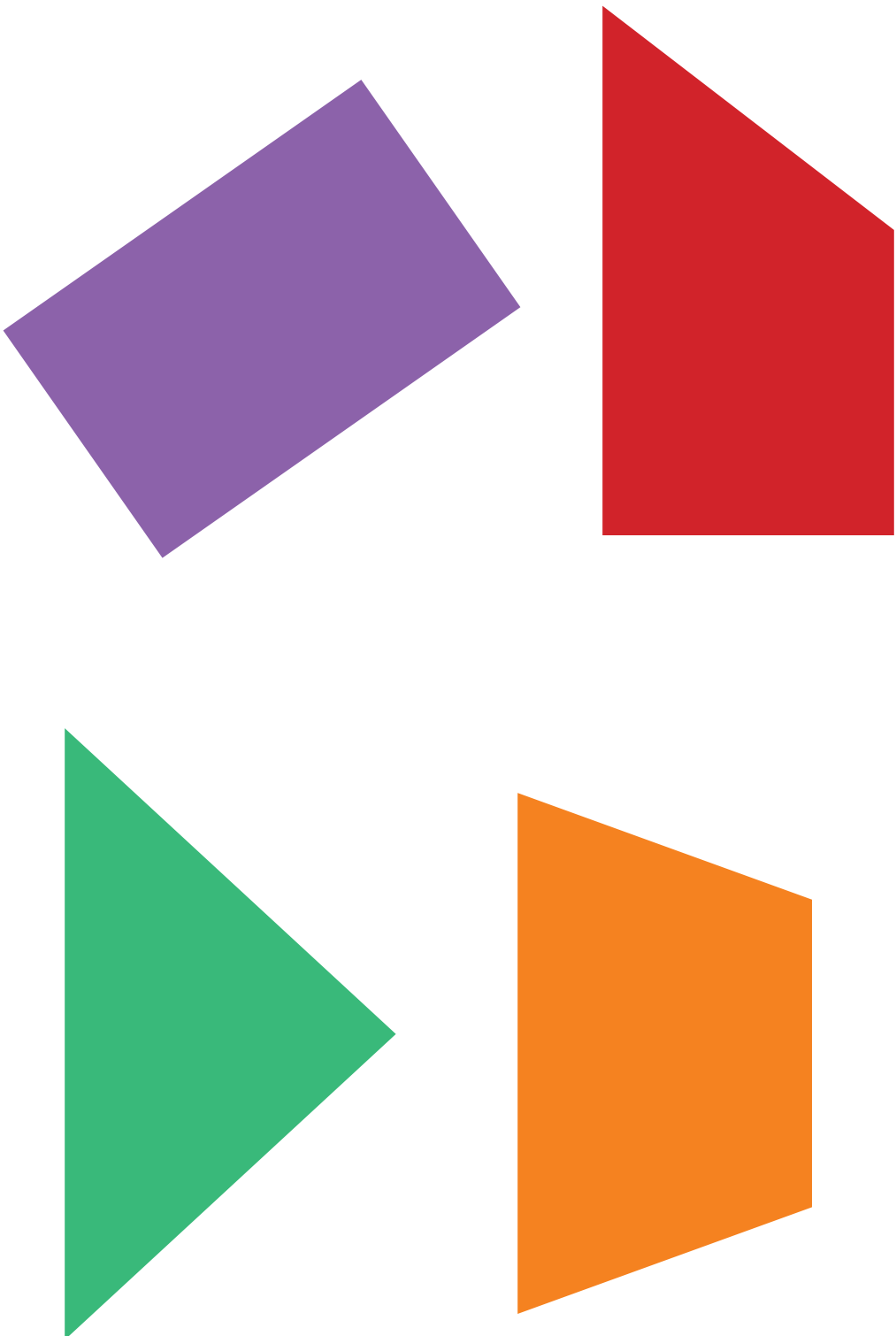
Materials available

none

Correct response

Purple, green, and orange triangles

Which Shape?

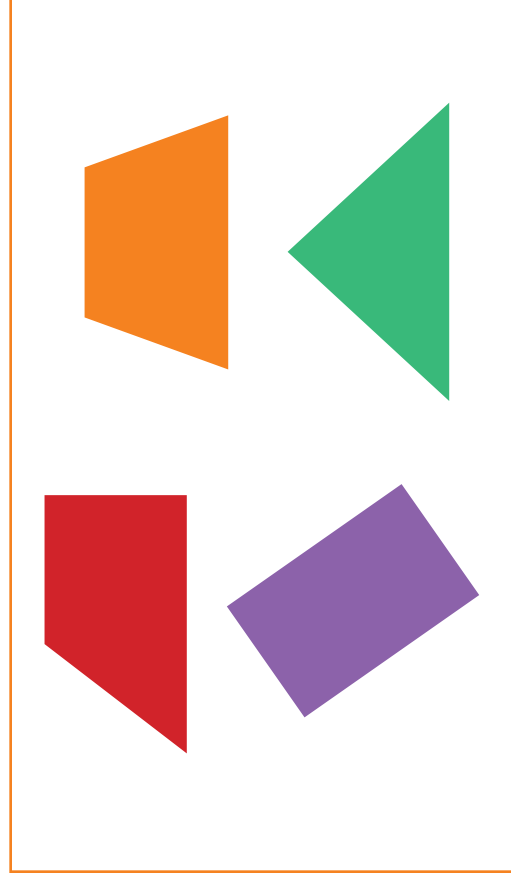


Which Shape?

- Say, “Listen to all of my clues and then tell me which one of these is my shape:”
 - “You can find square corners (right angles) in my shape.”
 - “The sides of my shape are all different lengths.”
 - “My shape has four sides.”

For further diagnosis

- Ask the student how he or she determined the shape.
- Does the student use all three clues?
- Does the student recognize that the trapezoids have four sides?
- Does the student use the angles clue?
- Does the nonstandard orientation of the rectangle affect the student's thinking?



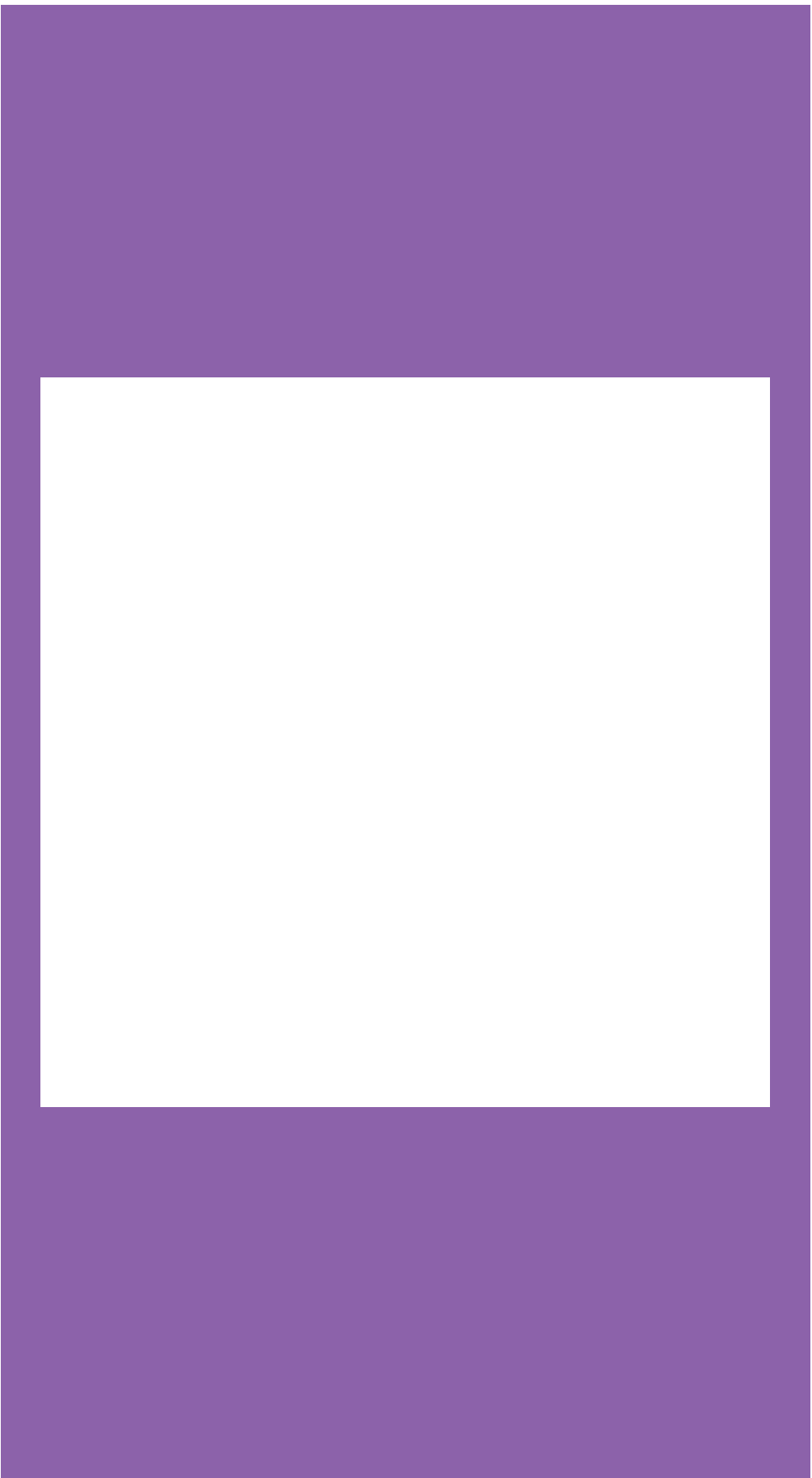
Materials available

none

Correct response

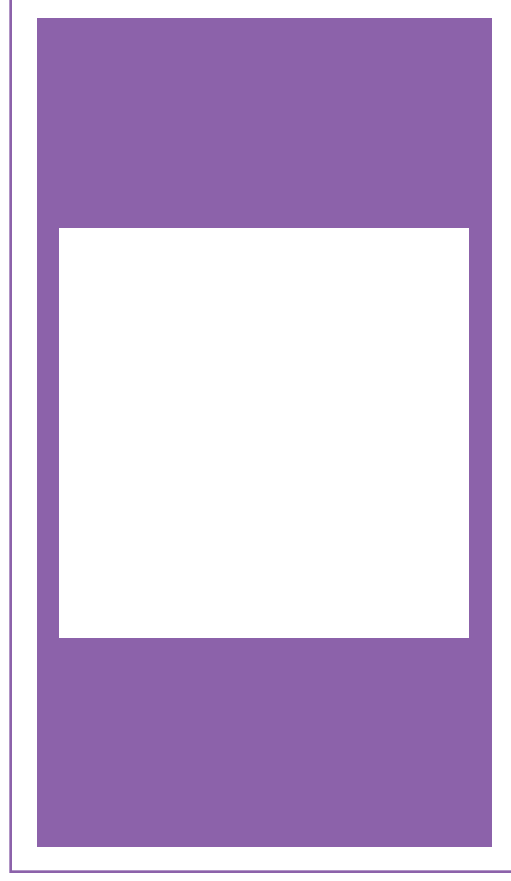
Right trapezoid (red shape)

Fill the White Space



Fill the White Space

- Say, “Can you fill the white space with these pieces?”
- If the student is struggling say, “Start with the large triangle here” and put the large triangle in place.
- If the student solves it with this help, score p (*partially correct*).



Materials available



prepared triangles

Correct response



Fills space exactly (in any orientation)

For further diagnosis

- Does the student use right angles as references for the corners?
- Does the student overlap pieces?
- Does the student rotate pieces until they fit?
- Does the student notice that the two small triangles can be put together to form the larger triangle?

Make a Square

Make a Square

- Hand the student four triangles.
- Say, “Use all four of these pieces to make a square.”
- After several seconds say, “Start by using two pieces to make a rectangle.”
- If the student solves it with this help, score p (*partially correct*).

For further diagnosis

- Does the student use right angles as references for the corners?
- Does the student find equal length segments and match them?
- Does the student form two rectangles?
- Does the student form the isosceles triangle in the middle first?



Materials available



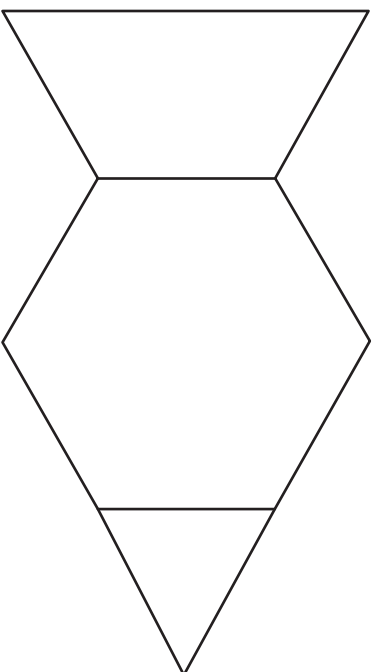
prepared triangles

Correct response

Forms a square using 4 right triangles
(in any orientation)

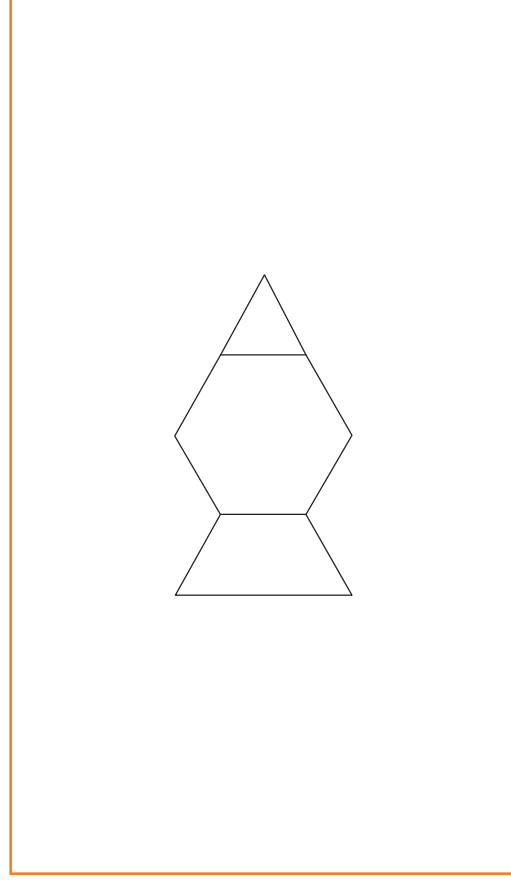


Fill the Frame



Fill the Frame

- Hand the student a supply of pattern blocks.
- Say, “Fill the frame with three blocks—one red, one yellow, and one green.”
- Say, “Now trade some pieces so that you fill the same space using a total of six blocks.”



Materials available

Correct response

Fills space using 6 blocks

For further diagnosis

- Does the student approach the task randomly or with purposeful trades?
- Ask the student to place one of each block on the table and cover each one as many different ways as possible. Leave these on the table.
- Now have the student return to the task and use the reference blocks to complete the task.

End of Concept Area 9



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