A Guide for

Assessing Mathematical Understanding

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A Guide for Assessing Mathematical Understanding

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Preface

Purpose

Early, high-quality instruction in mathematics that focuses on conceptual understanding, procedural fluency, and the ability to solve problems is of critical importance. Identifying what young students know about key mathematical ideas is the first step in planning high-quality lessons that meet the needs of all students. Assessing Mathematical Understanding is a collection of assessment tools developed to give teachers a way to access student thinking. This assessment can be used to track student progress, identify particular difficulties, and generally inform instructional planning. By using the assessment process described in this guide and understanding the learning framework on which it is based, teachers can increase their knowledge of how students learn. Using students' responses to the assessment items as a guide, teachers can differentiate instruction and create learning environments that better support their students' mathematical development.

Rationale

According to the latest available mathematics achievement data (U.S. Department of Education, National Center for Education Statistics [n.d.]), 59 percent of Northwest grade 4 public school students were not proficient in math. This rate is slightly better than the rate for the nation as a whole: 62 percent of students were not proficient across the U.S. (U.S. Department of Education, National Center for Education Statistics [n.d.]).

Such statistics underscore the need for early identification of students' mathematical misconceptions and holes in their conceptual knowledge. Addressing these problems will allow the teacher to act quickly to shore up areas of difficulty. Instructional strategies can be implemented to assist these children before they lose confidence in their ability to succeed in mathematics, thereby shrinking or preventing an achievement gap between these students and their mathematically proficient peers.

Applications

The assessments contained in Assessing Mathematical Understanding should be administered by an adult in a one-on-one interview format. The two grade-level assessments, *Kindergarten Items for Assessing Mathematical Understanding* and *First-Grade Items for Assessing Mathematical Understanding,* may be administered by a

teacher or paraprofessional. Each grade-level assessment is divided into three parts intended to be used three times in the year, with each section addressing two or three concept areas.

The comprehensive diagnostic assessment, *Diagnostic Items for Assessing Mathematical Understanding,* can be used to acquire extensive information about a student's understanding of one particular mathematical concept area. This assessment should be administered by a teacher who has an understanding of the mathematics concepts identified at these grade levels and beyond. A wealth of important instructional information can be gleaned by observing the student and asking probing questions about the student's thinking and reasoning about each task.

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Introduction

Description

A Guide for Assessing Mathematical Understanding provides an introduction to the Assessing Mathematical Understanding approach to helping teachers make informed decisions about their mathematics instruction. This book offers background, as well as directions, for using items in the collection, which includes *Kindergarten Items for Assessing Mathematical Understanding*, *First-Grade Items for Assessing Mathematical Understanding*, and *Diagnostic Items for Assessing Mathematical Understanding*, spanning learning goals for both of these grades.

The assessments provide both cumulative data about students' mathematical progress over time and in-depth diagnostic information. Assessing Mathematical Understanding is based on the belief that all students can succeed in mathematics when they have access to high-quality mathematics instruction. Each assessment is administered in a one-on-one interactive interview using a few simple classroom materials, including linking cubes, paper clips, base-ten blocks, paper, pattern blocks, and a pencil and ruler.

> "Young children love to think mathematically. They become exhilarated by their own ideas and the ideas of others. To develop the whole child, we must develop the mathematical child."—Clements & Sarama, 2009, p. 2

Grade-level assessments. Grade-level assessments for kindergarten and first grade are designed to be administered to each child by the teacher or another qualified individual such as a paraprofessional or support staff who has received training in conducting the assessments. These assessments can be administered two or three times during the year to document the student's progress. The student record (see page 54 for the kindergarten version) provides a cumulative record of the student's growth in mathematical proficiency during the course of the school year. The class record (see page 63 for the first-grade version) allows the teacher to document and see at a glance the progress of the entire class.

Diagnostic assessment. The diagnostic assessment provides in-depth assessment data. The teacher may choose to use the diagnostic assessment with students whom she believes would benefit from additional mathematical challenges or students who may be struggling with mathematical concepts. For each of these students, the teacher can identify a particular concept area and administer the bank of items in that section of the diagnostic assessment. The diagnostic interview yields detailed information about the student's mathematical knowledge and helps the teacher decide how to adjust curriculum and instruction to meet the needs of that child.

"The entire assessment gave me useful information. I saw all sorts of things about how my students solve problems and where they are confident."
—Teacher and field test participant, Helena School District, Montana

Goals

The primary goal of Assessing Mathematical Understanding is to enhance teachers' ability to meet individual student needs. By using these assessments and understanding the learning framework on which they are based, teachers can increase their knowledge of how students learn. Using students' responses to the assessment items as a guide, teachers can differentiate instruction and create learning environments that better support their students' mathematical development.

"Because instruction should be based on what each child knows, it is essential to assess continually whether a child could solve a particular problem and how he or she solves it."—Baroody & Standifer, 1993, p. 80

A second goal is to facilitate teacher collaboration around student learning. When used by all members of a K–1 teaching team, Assessing Mathematical Understanding provides a common tool that can stimulate discussions about mathematical learning in general and about particular trouble spots for students. These grade-level or cross-grade discussions can promote shared responsibility among teachers for addressing troublesome content at one or multiple grade levels. Collaborative and consistent use of the assessments by teaching teams will increase the impact of their use.

"The overall strength of the assessment is that this material provides a common framework for analyzing individual student results."
—Assessment specialist and reviewer, Portland, Oregon

A third goal is to promote student learning in mathematics. Participation in one-on-one assessment interviews benefits students because in this individualized format they can show and tell their reasoning and strategies to an attentive adult. This kind of personal interaction about mathematics helps young children clarify their own understanding of mathematical ideas. Ultimately, as teachers regularly provide instruction tailored to individual needs, students will find increased success in mathematics and feel appropriately challenged in their learning. This, in turn, contributes to students' self-efficacy and positive attitudes toward learning mathematics.

Background

Importance of early learning experiences

Assessing Mathematical Understanding was developed in response to research and recommendations emphasizing the importance of early learning experiences in mathematics. Concerns about the mathematical performance of American children consistently point to the significant role of early, high-quality instruction in mathematics that focuses on conceptual understanding, procedural fluency, and the ability to solve problems using mathematical understanding.

> "Children have an impressive, often untapped, potential to learn mathematics. For many this has been a potential largely left unrealized. It is not their developmental limitation, but a limitation of the society and its schools."
> —Sarama & Clements, 2009, p. 25

The National Council of Teachers of Mathematics (NCTM), the National Association for the Education of Young Children (NAEYC), and the National Mathematics Advisory Panel (NMAP) all affirm that high-quality, challenging, and accessible mathematics education for young children is a vital foundation for future mathematics learning (National Association for the Education of Young Children [NAEYC] & the National Council of Teachers of Mathematics [NCTM], 2010; NCTM, 2000; National Mathematics Advisory Panel [NMAP], 2008).

Attention to the mathematical development of young children has benefits beyond improved mathematics performance. Early knowledge strongly affects later success in mathematics (Sarama & Clements, 2009). Research has shown that early mathematics skills have greater predictive power for future school achievement than reading or attention skills (Clements & Sarama, 2009; Duncan et al., 2007).

However, there are significant differences in the numerical knowledge of children in the early years of school, and these differences increase as children progress through school (Clements & Sarama, 2009; Wright, Martland, & Stafford, 2006). Children who are low performers at the beginning of school tend to remain so, and the gap between them and higher performers tends to increase over time. Compounding the problem, low-performing children begin to develop strong negative attitudes toward mathematics because they lack experiences of success in school mathematics (NMAP, 2008; Wright et al., 2006).

> "A positive attitude toward mathematics and a strong foundation for mathematics learning begin in early childhood."—NAEYC & NCTM, 2010, p. 18

To prevent a long-term negative effect, it is important that teachers identify children who are at risk of not learning mathematics successfully as early as possible in their schooling (Clarke, Gervasoni, & Sullivan, 2000). Early identification of students' mathematical misconceptions and holes in their conceptual knowledge allows the teacher to act quickly to shore up areas of difficulty. Instructional strategies can be implemented to assist these children before they lose confidence in their ability to succeed in mathematics, thereby shrinking or preventing an achievement gap between these students and their mathematically proficient peers.

Assessing Mathematical Understanding is intended to assist teachers in identifying children who are at risk of not learning mathematics successfully and who may benefit from additional assistance. Early intervention provides an opportunity to reduce the performance gap before educationally disadvantaged children experience too much failure. Furthermore, early interventions have been shown to prevent later learning difficulties in school (Clements, 2004; Dowker, 2004, as cited in Wright et al., 2006).

One-on-one interviews

A one-on-one interview provides considerable insight into what children know and can do. Interviews allow the teacher to engage in conversation with each child to determine the extent of his or her knowledge and the relative sophistication of the child's numerical strategies. By asking probing questions, teachers can encourage a student to clarify his or her interpretation of both the problem and his response. During the interview, a teacher gathers data about problem-solving strategies and thinking processes students use to approach each problem. By interviewing many students, teachers develop awareness of common misconceptions and the range of strategies that many students in the class possess.

> "The levels at which problems are solved are more important than the nature of the problems themselves."—Van den Heuvel-Panhuizen & Senior, 2001, p. 17

By using one-on-one mathematics interviews, teachers can learn a great deal about students and can identify previously undiscovered capabilities in students. The individual interview can provide insight about children who might be reluctant to talk in a group but who have a great deal of mathematical understanding. Teachers report that quiet achievers—many of them girls—emerge through one-on-one interviews. These quiet achievers might not speak up during discussion in the classroom, but given the time to work one-on-one with an adult, they reveal what they can do (Clarke, Mitchell, & Roche, 2005).

Teachers also report that individual interviews offer surprising insights about their higher achieving students. One teacher who participated in the field test of Assessing Mathematical Understanding said, "I thought I was doing a great job with my bright students, but some of them were having trouble. I need to let them work more with manipulatives."

Formative assessment

The word *assessment* comes from the Latin *assidere*, which literally means "to sit beside." As the root implies, assessment should be an interactive process that serves as the bridge between teaching and learning and helps teachers adjust instruction to better meet student needs (Wiliam, 2007). Formative assessments should illuminate student strengths and weaknesses in order to help teachers adjust instruction accordingly. Classroom interactions and day-to-day observations about student learning are types of formative assessment that can reveal children's thinking and give the teacher a picture of students' mathematical strength and needs (Lindquist & Joyner, 2004). In addition, a formal assessment such as Assessing Mathematical Understanding

can also serve a formative function. When this assessment is administered to individual students periodically throughout the year, teachers have another source of formative data that can be used to inform instruction.

Assessment is a crucial element in effective teaching and can enhance students' learning as well as measure it (NCTM, 2000; NMAP, 2008). A range of data sources can be used to identify each child's unique strengths and needs so as to inform instructional planning. Beginning with careful observation, assessment should use multiple sources of information systematically gathered over time (NAEYC & NCTM, 2010). Assessing Mathematical Understanding is intended to be a formative assessment tool that serves as one of these sources of information.

> "The greatest value in formative assessment lies in teachers and students making use of results to improve real-time teaching and learning at every turn."—Chappuis & Chappuis, 2008, p. 17

The NCTM and NAEYC assert that in high-quality mathematics education for young children, teachers and other key professionals support children's learning by thoughtfully and continually assessing all students' mathematical knowledge, skills, and strategies (NAEYC & NCTM, 2010). However, without a systematic focus on student progress in mathematics, teachers are not able to identify children's needs in this area in the critical early years of schooling (Wright et al., 2006). A suitable assessment instrument such as Assessing Mathematical Understanding is critical to implementation of this process.

When teachers document student progress over time using formative assessments, they create profiles of students' mathematical learning. These profiles show not only how far the students have come but also offer guidance for how the instruction might proceed.

> "Diagnostic tools are required to identify the specific problems children are experiencing and to profile strengths and weaknesses. The tools should also indicate children's particular misconceptions and incorrect strategies."
> —Wright et al., 2006, p. 1

Potential misuses of assessment

Assessing Mathematical Understanding is intended to be used as a tool to track student progress, identify particular difficulties, and generally inform instructional planning. Care must be taken that the assessments are not used in inappropriate ways that might limit any child's access to high-quality and challenging instruction. The NAEYC and NCTM mathematics position statement includes this caution: "Educators must take care that assessment does not narrow the curriculum and inappropriately label children. If assessment results exclude some children from challenging learning activities, they undercut educational equity." (NAEYC & NCTM, 2010, p. 13).

Furthermore, learning goals for Assessing Mathematical Understanding should not be viewed as a checklist of isolated skills. Teachers should recognize that knowledge is integrated and never simply emphasize the learning of unrelated facts and skills. The aim of instruction should be to equip students to use their mathematical skills and insights to solve a whole range of problems from both daily life and the world of mathematics (Van den Heuvel-Panhuizen & Senior, 2001).

Instructional practices

The teacher is the most important component in the instructional environment. Teachers, rather than textbooks or other instructional materials, have the capacity to understand the progression of student learning and respond to individual student needs at the appropriate time. Students' understanding of mathematics, their ability to use it to solve problems, and their confidence in doing mathematics are all shaped by the teaching they encounter in school.

> "The debate about mathematics reform has focused primarily on curriculum, not on professional development or instruction. … Yet this research review suggests that in terms of outcomes on traditional measures, such as standardized tests and state accountability assessments, curriculum differences appear to be less consequential than instructional differences."
> —Slavin & Lake, 2007, p. 39

Following the administration of grade-level or diagnostic versions of Assessing Mathematical Understanding, teachers should use the results for the class and for individual students to adjust instruction to meet individual needs.

> "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well."—NCTM, 2000, p. 16

Provide an engaging learning environment. Learning mathematics is a sense-making activity. The classroom structure and practices should help children give meaning to numbers and numerical facts in everyday life and to deal with them appropriately. The instructional environment that the teacher creates should ensure that children are able to explore number relationships and identify strategies that reliably lead to solutions. A dynamic learning environment for mathematics is one in which students are engaged in activities and conversations about mathematics and have a variety of tools for learning readily available.

Primary-grade mathematics classrooms should be characterized by opportunities to discover, explore, invent, and discuss mathematical concepts through well-planned, intentionally sequenced lessons. Rather than being dominated by "teacher telling," instruction should include questions and invitations to create and test strategies and solutions to interesting problems.

> "… I point to the proactive role of the teacher in establishing an appropriate classroom culture, in choosing and introducing instructional tasks, organizing group work, framing topics for discussion, and orchestrating discussion."
> —Gravemeier, 2004, p. 126

Discussing problem solutions in a trusting environment leads to opportunities for conceptual development and alternative thinking strategies. Such an environment has been shown to be conducive to students' development of positive beliefs about mathematics and their ability to solve problems (Fuson, Kalchman, & Bransford, 2005; NCTM, 2000; Van de Walle & Watkins, 1993).

Encourage meaningful academic conversation. Classroom interactions between the teacher and students, as well as among students, are critical to students' learning (Van den Heuvel-Panhuizen & Senior, 2001). Students benefit from peer feedback on their ideas. Discussions with their peers cause students to reflect on their own ideas and thus strengthen their understanding of mathematical relationships.

The Common Core State Standards for Mathematical Practice call for students to "justify their conclusions, communicate them to others, and respond to the arguments of others" (Common Core State Standards

Initiative [CCSSI], 2010, pp. 6–7). Such instructional conversations should be directed by the teacher, but should include a high degree of child-to-child talk. When children discuss and compare varied approaches to problems they can increase their success with problem solving and gain satisfaction from the sharing and validation of the methods they develop.

> "One important way to make students' thinking visible is through math talk—talking about mathematical thinking. … Such communication about mathematical thinking can help everyone in the classroom understand a given concept or method because it elucidates contrasting approaches, some of which are wrong—but for interesting reasons."—Fuson et al., 2005, p. 228

Use a problem-centered approach to instruction. A problem-centered approach helps children develop understanding of mathematical concepts. Each student should be able to relate problems, concepts, or skills being learned to the knowledge that he or she already possesses. When teachers pose problems and tasks of varied types, they promote the development of new strategies. When a student encounters a problem that is just a little more difficult than the ones he can readily solve, he must apply either a new strategy or a combination of strategies to the new, more complex task. Over time, this results in the child developing a rich repertoire of problem-solving strategies. In order to know where students are in their thinking and what strategies they are able to use in which situations, teachers must observe, discuss, and ask questions while students are solving problems.

The first Common Core State Standard for Mathematical Practice states that teachers should develop students' ability to "make sense of problems and persevere in solving them" (CCSSI, 2010, p. 6).

Word problems provide a strong foundation for students' conceptual understanding of numbers, operations, and relationships between numbers. When students encounter mathematical concepts through word problems set in familiar situations, they are able to use the context of the problems to make sense of mathematical relationships and to develop their own strategies for finding the answers. Although we often think of word problems as being more difficult to solve than basic arithmetic problems, the reverse is actually true. Research tells us that when children are learning about numbers and operations they benefit from problems in contexts that help them to conceptualize and model situations (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fuson et al., 2005; Kilpatrick, Swafford, & Findell, 2001; Van de Walle, 2004).

Maintain high cognitive demand in instruction. Children in the primary grades are capable of a great deal of mathematical thought (Carpenter et al., 1999). Children should routinely be engaged in thinking hard to solve numerical problems which they find quite challenging. Rather than spending a great deal of instructional time on routine calculations and procedural knowledge, teaching will be most effective when the content is focused just beyond the child's current knowledge level. When tasks posed to children slightly exceed their present level of understanding, they must actively engage in reformulating the problem or their solution strategy and then reflect on whether they have solved the original problem or need to engage in more thinking. Over time, this cycle moves children to new levels of thinking (Sarama & Clements, 2009).

Teachers should provide children with sufficient time to solve problems. This means that lessons will include frequent opportunities for students to engage in sustained thinking, reflection on their thinking, and reflecting on the results of their thinking. Furthermore, instruction should underscore that there can be more than one correct method for finding solutions to any problem (Baroody & Standifer, 1993). By exploring connections among multiple solution methods, students' mathematical understanding is strengthened.

Understand and respond to natural progressions in student learning. In the mathematical development of young children variation is the norm, not the exception. However, children do tend to follow similar sequences or learning trajectories as they gain mathematical understanding (Sarama & Clements, 2009). Teachers must recognize that learning is neither linear nor strictly sequenced and that there is a stratified nature to the learning process. Teachers should understand children's numerical strategies and the typical progression of those strategies and plan instruction accordingly (Carpenter et al., 1999; Sarama & Clements, 2009).

"Research indicates that knowledge gaps appeared in large part due to the lack of connection between children's informal and intuitive knowledge and school mathematics. … High quality experiences in early mathematics can ameliorate such problems."—Sarama & Clements, 2009, p. 6

The ways that young children interpret and make sense of mathematical ideas are different from those of adults (Carpenter et al., 1999; Sarama & Clements, 2009; Van de Walle, 2004). When teachers assume that children "see" situations, problems, and mathematical contexts in the same way as an adult, their attempt to help a student by providing an explanation that makes sense to them has the potential to confuse the student and interfere with his learning. Teaching should support and build on students' intuitive strategies because these form the basis for development of more sophisticated strategies later.

"Follow the natural developmental progression when selecting new knowledge to be taught. By selecting learning objectives that are a natural next step … the teacher will be creating a learning path that is developmentally appropriate for children, one that fits the progression of understanding as identified by researchers."—Griffin, 2005, p. 266

Concept Areas and Learning Goals

Organizing principles

Assessing Mathematical Understanding was developed in response to research and recommendations emphasizing the importance of early learning experiences in mathematics. Assessing Mathematical Understanding is organized around the following nine concept areas with 51 corresponding learning goals.

Concept Area 1: Verbal Counting
1.1) Counts by ones
1.2) Counts forward from variable starting points
1.3) Counts backward by ones from variable starting points
1.4) Uses skip counting

Concept Area 2: Counting Objects
2.1) Counts objects in a given collection
2.2) Produces a collection of a specified size
2.3) Recognizes collections arranged in patterns without counting
2.4) Writes the numeral to represent a quantity

Concept Area 3: Adding to and Taking From in Contexts
3.1) Solves context problems of the type join, result unknown
3.2) Solves context problems of the type separate, result unknown
3.3) Solves context problems of the type part-part-whole, whole unknown
3.4) Solves context problems of the type part-part-whole, part unknown
3.5) Solves context problems of the type separate, change unknown
3.6) Solves context problems of the type compare, difference unknown
3.7) Solves context problems of the type separate, start unknown
3.8) Solves context problems of the type join, start unknown

Concept Area 4: Comparing and Ordering Numbers
4.1) Compares sets or numbers
4.2) Orders 3 or more numbers
4.3) Represents numbers on the number line
4.4) Identifies ordinal position
4.5) Determines how many more or less

Concept Area 5: Fluency With Number Combinations
5.1) Composes and decomposes numbers
5.2) Knows "plus 1" is next counting word
5.3) Knows addition combinations for doubles
5.4) Knows addition combinations for near doubles
5.5) Knows addition combinations based on 10
5.6) Knows other addition combinations
5.7) Knows "minus 1" is the previous counting word
5.8) Knows subtraction combinations for doubles
5.9) Knows subtraction combinations for near doubles
5.10) Knows subtraction combinations based on 10
5.11) Knows other subtraction combinations

Concept Area 6: Properties and Symbols
6.1) Translates between word problems and number sentences
6.2) Identifies the connection between addition and subtraction and counting forward and backward
6.3) Compares numbers using symbols
6.4) Recognizes and uses properties of addition
6.5) Recognizes addition-subtraction complement and inverse principle

Concept Area 7: Place Value
7.1) Recognizes base-ten equivalents
7.2) Translates among place value models, count words, numerals
7.3) Reads and writes multidigit numbers meaningfully
7.4) Decomposes a larger unit into smaller units by place value
7.5) Adds multidigit whole numbers
7.6) Subtracts multidigit whole numbers

Concept Area 8: Measurement
8.1) Makes comparisons based on measurable attributes
8.2) Measures length
8.3) Understands units

Concept Area 9: Geometry
9.1) Identifies quadrilaterals in standard orientation
9.2) Identifies triangles in standard orientation
9.3) Identifies geometric figures in nonstandard orientation
9.4) Identifies components and properties of shapes
9.5) Composes geometric figures

It is important to emphasize that the 51 learning goals should not be regarded as isolated skills. Many mathematical concepts are closely integrated in nature, and assessing student understanding about one concept provides information about understanding of others (Van den Heuvel-Panhuizen & Senior, 2001).

Common Core State Standards

The grade-level assessments support teachers by providing information on the progress students are making in areas aligned with the Common Core State Standards (CCSS) for Mathematics (CCSSI, 2010). The assessment items address the mathematical concepts and skills identified in the CCSS for kindergarten and first grade for all of the Common Core domains: counting and cardinality, operations and algebraic thinking, number and operations in base ten, measurement and data, and geometry.

Table 1 outlines the CCSS for Mathematics in kindergarten and first grade and how they correspond to Assessing Mathematical Understanding concept areas and learning goals.

Table 1. Alignment of Assessing Mathematical Understanding Concept Areas and Learning Goals With CCSS for Mathematics

CCSS for Mathematics "Domains" and "Cluster Headings" for Kindergarten and First Grade	Assessing Mathematical Understanding "Concept Areas" and "Learning Goals"
Counting and Cardinality • Know number names and the count sequence • Count to tell the number of objects • Compare numbers	**Concept Area 1: Verbal Counting** 1.1) Counts by ones 1.2) Counts forward from variable starting points 1.3) Counts backward by ones from variable starting points 1.4) Uses skip counting **Concept Area 2: Counting Objects** 2.1) Counts objects in a given collection 2.2) Produces a collection of a specified size 2.3) Recognizes collections arranged in patterns without counting 2.4) Writes the numeral to represent a quantity **Concept Area 4: Comparing and Ordering Numbers** 4.1) Compares sets or numbers 4.2) Orders 3 or more numbers 4.3) Represents numbers on the number line 4.4) Identifies ordinal position 4.5) Determines how many more or less
Operations and Algebraic Thinking • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from • Represent and solve problems involving addition and subtraction • Understand and apply properties of operations and the relationship between addition and subtraction • Add and subtract within 20 • Work with addition and subtraction equations	**Concept Area 3: Adding to and Taking From in Contexts** 3.1) Solves context problems of the type join, result unknown 3.2) Solves context problems of the type separate, result unknown 3.3) Solves context problems of the type part-part-whole, whole unknown 3.4) Solves context problems of the type part-part-whole, part unknown 3.5) Solves context problems of the type separate, change unknown 3.6) Solves context problems of the type compare, difference unknown 3.7) Solves context problems of the type separate, start unknown 3.8) Solves context problems of the type join, start unknown **Concept Area 6: Properties and Symbols** 6.1) Translates between word problems and number sentences 6.2) Identifies the connection between addition and subtraction and counting forward and backward 6.3) Compares numbers using symbols 6.4) Recognizes and uses properties of addition 6.5) Recognizes addition-subtraction complement and inverse principle

CCSS for Mathematics "Domains" and "Cluster Headings" for Kindergarten and First Grade	Assessing Mathematical Understanding "Concept Areas" and "Learning Goals"
Number and Operations in Base Ten • Work with numbers 11–19 to gain foundations for place value • Extend the counting sequence • Understand place value • Use place value understanding and properties of operations to add and subtract	**Concept Area 5: Fluency With Number Combinations** 5.1) Composes and decomposes numbers 5.2) Knows "plus 1" is next counting word 5.3) Knows addition combinations for doubles 5.4) Knows addition combinations for near doubles 5.5) Knows addition combinations based on 10 5.6) Knows other addition combinations 5.7) Knows "minus 1" is the previous counting word 5.8) Knows subtraction combinations for doubles 5.9) Knows subtraction combinations for near doubles 5.10) Knows subtraction combinations based on 10 5.11) Knows other subtraction combinations **Concept Area 7: Place Value** 7.1) Recognizes base-ten equivalents 7.2) Translates among place value models, count words, numerals 7.3) Reads and writes multidigit numbers meaningfully 7.4) Decomposes a larger unit into smaller units by place value 7.5) Adds multidigit whole numbers 7.6) Subtracts multidigit whole numbers
Measurement and Data • Describe and compare measurable attributes • Classify objects and count the number of objects in categories* • Measure lengths indirectly and by iterating length units • Tell and write time* • Represent and interpret data	**Concept Area 8: Measurement** 8.1) Makes comparisons based on measurable attributes 8.2) Measures length 8.3) Understands units
Geometry • Identify and describe shapes • Analyze, compare, create, and compose shapes • Reason with shapes and their attributes	**Concept Area 9: Geometry** 9.1) Identifies quadrilaterals in standard orientation 9.2) Identifies triangles in standard orientation 9.3) Identifies geometric figures in nonstandard orientation 9.4) Identifies components and properties of shapes 9.5) Composes geometric figures

*There are no Assessing Mathematical Understanding assessment items for these cluster headings.

Organizational Framework

Concept areas

> "Knowledge of developmental progressions—levels of understanding and skill, each more sophisticated than the last—is essential for high-quality teaching based on understanding both mathematics and children's thinking and learning."—Sarama & Clements, 2009, p. 17

Learning progressions describe children's thinking as they develop mathematical understanding. They provide teachers with reference points and benchmarks that can inform how to plan instruction that will move students forward in their mathematical learning (Dodge, Heroman, Charles, & Maiorca, 2004; Sarama & Clements, 2009). The organizational framework in *A Guide for Assessing Mathematical Understanding* describes general progressions and landmarks in learning.

Assessing Mathematical Understanding is organized around nine concept areas:

1) Verbal Counting
2) Counting Objects
3) Adding to and Taking From in Contexts
4) Comparing and Ordering Numbers
5) Fluency With Number Combinations
6) Properties and Symbols
7) Place Value
8) Measurement
9) Geometry

Frameworks

Each of the nine concept areas includes learning goals that describe incremental levels in development and performance levels of increasing conceptual demand. Table 2 illustrates the components of the framework. Learning goals are listed in the left column. Performance levels are indicated by A, B, and C in the top row. Within each cell is the expected content limit or boundary for student performance for each learning goal.

Table 2. Sample Framework

Concept Area 1: Verbal Counting

		Performance level		
Learning goal		A	B	C
1.1	Counts by ones	To 10	To 20	To at least 40
1.2	Counts forward from variable starting points	Start value less than 10	Start value in teens	Start value above 50
1.3	Counts backward by ones from variable starting points	***From 10***	***From 20***	***Start value above 50 (across decades)***
1.4	Uses skip counting	***By tens to 100***	***By fives to 55***	***By twos to 24***

Content limit

Blue items are above grade level and appear in the diagnostic assessment

There is an assessment item associated with each content limit in the concept area tables. The purple cells correspond to kindergarten learning goals, orange cells to first-grade learning goals, and the blue cells are considered above first grade. All cells with content limits have an item in the diagnostic assessment. A subset of these items are in a grade-level assessment. The content limits in ***bold*** and ***italic*** in the table are not included in a grade-level assessment. For example, 1.3A is a kindergarten learning goal, with an assessment item that is only found in the diagnostic assessment. The learning goal 1.2C has an assessment item in the diagnostic assessment as well as the first-grade assessment. Assessment items associated with 1.4B and 1.4C are only in the diagnostic assessment and are also considered above first grade level. Kindergarten items are purple (K ■), first-grade items are orange (F ■), and above first-grade items are blue (above F ■).

Each framework is accompanied by a list of competencies describing the big ideas contained within the concept area (see Tables 3–11). Teachers can use these competency statements as the basis for further data collection through class observations and follow-up instructional tasks.

Competencies for Verbal Counting

Does the student understand and use these big ideas?

- Numbers belong in a sequence that includes various patterns
- Familiarity with the number sequence allows us to count forward or backward from any point in the sequence
- Knowing number patterns allows us to count by numbers other than one

Table 3. Framework for Concept Area 1: Verbal Counting

Concept Area 1: Verbal Counting

		Performance level		
Learning goal		A	B	C
1.1	Counts by ones	To 10	To 20	To at least 40
1.2	Counts forward from variable starting points	Start value less than 10	Start value in teens	Start value above 50
1.3	Counts backward by ones from variable starting points	***From 10***	***From 20***	***Start value above 50 (across decades)***
1.4	Uses skip counting	***By tens to 100***	***By fives to 55***	***By twos to 24***

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Competencies for Counting Objects

Does the student understand and use these big ideas?

- A collection of objects can be counted to find out how many are in it
- Counting by numbers other than one allows us to count faster and more reliably
- Some collections are easily recognizable and we know how many are included in them without counting

Table 4. Framework for Concept Area 2: Counting Objects

Concept Area 2: Counting Objects

		Performance level		
Learning goal		A	B	C
2.1	Counts objects in a given collection	To 10	To 20	***To 100 with objects grouped in tens***
2.2	Produces a collection of a specified size	To 10	To 20	***To 100***
2.3	Recognizes collections arranged in patterns without counting	***To 4***	***To 6***	***To 10***
2.4	Writes the numeral to represent a quantity	To 10	To 20	To 100

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Competencies for Adding to and Taking From in Contexts

Does the student understand and use these big ideas?

- Physical objects or drawings can be used to solve problems involving joining, separating, comparing, and considering parts of a whole collection
- Counting forward or backward can be used to solve problems involving joining, separating, comparing, and considering parts of a whole collection
- Number relationships and known number facts can be used to solve problems involving joining, separating, comparing, and considering parts of a whole collection

Table 5. Framework for Concept Area 3: Adding to and Taking From in Contexts

Concept Area 3: Adding to and Taking From in Contexts

	Performance level		
Learning goal	A	B	C
3.1 Solves context problems of the type join, result unknown (JRU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.2 Solves context problems of the type separate, result unknown (SRU)	Totals 2 to 6	Totals 7 to 10	Totals 11 to 18
3.3 Solves context problems of the type part-part-whole, whole unknown (PPW-WU)	***Totals 2 to 6***	Totals 7 to 10	Totals 11 to 18
3.4 Solves context problems of the type part-part-whole, part unknown (PPW-PU)	***Totals 2 to 6***	Totals 7 to 10	Totals 11 to 18
3.5 Solves context problems of the type separate, change unknown (SCU)	***Totals 2 to 6***	Totals 7 to 10	Totals 11 to 18
3.6 Solves context problems of the type compare, difference unknown (CDU)	***Totals 2 to 6***	Totals 7 to 10	Totals 11 to 18
3.7 Solves context problems of the type separate, start unknown (SSU)	***Totals 2 to 6***	***Totals 7 to 10***	Totals 11 to 18
3.8 Solves context problems of the type join, start unknown (JSU)	***Totals 2 to 6***	***Totals 7 to 10***	Totals 11 to 18

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Competencies for Comparing and Ordering Numbers

Does the student understand and use these big ideas?

- The relationship among numbers can be described and represented in multiple ways
- Any group of numbers can be placed in order by magnitude even if they are not adjacent in the counting sequence
- Ordinal numbers tell the position of an element in a sequence

Table 6. Framework for Concept Area 4: Comparing and Ordering Numbers

Concept Area 4: Comparing and Ordering Numbers

	Performance level		
Learning goal	A	B	C
4.1 Compares sets or numbers	Sets to 10	Sets vs. numbers to 20	***Numbers to 100***
4.2 Orders 3 or more numbers	3 numbers less than 10	4 numbers to 20	5 numbers to 100
4.3 Represents numbers on the number line	***To 10, with grid marks***	***To 20, with grid marks***	***To 100, with only decade grid marks***
4.4 Identifies ordinal position	***To 10th***	***To 30th***	***Reads ordinal terms (words) through 9th and uses them***
4.5 Determines how many more or less	***By comparing sets (to 10)***	***By counting on (numbers to 20, differences within 5)***	***By adding or subtracting, to 100***

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Competencies for Fluency With Number Combinations

Does the student understand and use these big ideas?

- Numbers can be composed and decomposed in multiple ways without changing their values
- Known addition number combinations can be used to determine unknown combinations
- The relationship between addition and subtraction can be used to determine unknown subtraction combinations

Table 7. Framework for Concept Area 5: Fluency With Number Combinations

Concept Area 5: Fluency With Number Combinations

		Performance level		
Learning goal		A	B	C
5.1	Composes and decomposes numbers	Constructs partners, with objects, totals to 5	Constructs partners, with objects, totals to 10	
5.2	Knows "plus 1" is next counting word	***Totals 2 to 5***	***Totals 6 to 9***	
5.3	Knows addition combinations for doubles	Totals 2 to 5	***Totals 6 to 9***	***Totals 11 to 18***
5.4	Knows addition combinations for near doubles	Totals 2 to 5	***Totals 6 to 9***	***Totals 11 to 18***
5.5	Knows addition combinations based on 10		Totals equal to 10	***Combinations with 9***
5.6	Knows other addition combinations		Totals 6 to 9	***Totals 11 to 18***
5.7	Knows "minus 1" is previous counting word	***Totals 2 to 5***	***Totals 6 to 9***	
5.8	Knows subtraction combinations for doubles	Totals 2 to 5	***Totals 6 to 9***	***Totals 11 to 18***
5.9	Knows subtraction combinations for near doubles	***Totals 2 to 5***	Totals 6 to 9	***Totals 11 to 18***
5.10	Knows subtraction combinations based on 10		Totals equal to 10	***Combinations with 9***
5.11	Knows other subtraction combinations		Totals 6 to 9	***Totals 11 to 18***

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Within this concept area, the addition and subtraction number combinations have been organized by number value and by the relationships that tend to determine the order in which they are learned and their relative level of difficulty. For each number combination listed, all the related number combinations should be considered with it. For example, the number combination 5 + 2 = 7 includes 2 + 5 = 7, 7 – 5 = 2, and 7 – 2 = 5.

Number Combinations

Plus 1	Doubles	Near doubles	Combinations based on 10	Other combinations
1 + 1	1 + 1	1 + 2	1 + 9	2 + 4
1 + 2	2 + 2	2 + 3	2 + 8	2 + 5
1 + 3	3 + 3	3 + 4	3 + 7	2 + 6
1 + 4	4 + 4	4 + 5	4 + 6	2 + 7
1 + 5	5 + 5	5 + 6	5 + 5	3 + 5
1 + 6	6 + 6	6 + 7	2 + 9	3 + 6
1 + 7	7 + 7	7 + 8	3 + 9	3 + 8
1 + 8	8 + 8	8 + 9	4 + 9	4 + 7
1 + 9	9 + 9		5 + 9	4 + 8
			6 + 9	5 + 7
			7 + 9	5 + 8
			8 + 9	6 + 8

Competencies for Properties and Symbols

Does the student understand and use these big ideas?

- Number relationships can be expressed in both words and symbols
- Addition and subtraction are connected to counting forward and backward
- Using properties of numbers and operations can simplify calculations

Table 8. Framework for Concept Area 6: Properties and Symbols

Concept Area 6: Properties and Symbols

		Performance level		
Learning goal		A	B	C
6.1	Translates between word problems and number sentences	***JRU, totals 2 to 9***	SCU, totals 11 to 18	JSU, totals 11 to 18
6.2	Identifies the connection between addition and subtraction and counting forward and backward	Connects adding to counting on	***Connects subtracting to counting back***	
6.3	Compares numbers using symbols	***Using =***	***Using <, >***	***Produces symbol***
6.4	Recognizes and uses properties of addition		Commutative property	***Associative property to add 3 single-digit numbers***
6.5	Recognizes addition-subtraction complement and inverse principle	***Inverse principle, single-digit numbers in a context***	***Complement principle in a context***	***Inverse principle in symbols***

JRU = Join, result known
SCU = Separate, change unknown
JSU = Join, start unknown

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Competencies for Place Value

Does the student understand and use these big ideas?

- Items can be grouped or ungrouped according to place value units
- The position of a digit (in a multidigit number) indicates its value
- Computing with multidigit numbers can involve regrouping based on place value

Table 9. Framework for Concept Area 7: Place Value

Concept Area 7: Place Value

		Performance level		
Learning goal		A	B	C
7.1	Recognizes base-ten equivalents	***10 ones = 1 ten***	***10 tens = 100 ones = 1 hundred***	***10 hundreds = 1,000***
7.2	Translates among place value models, count words, numerals	Teens	2-digit numbers	***3-digit numbers***
7.3	Reads and writes multidigit numbers meaningfully	To 20	2-digit numbers	***3-digit numbers***
7.4	Decomposes a larger unit into smaller units by place value	***To 30***	2-digit numbers	***3-digit numbers***
7.5	Adds multidigit whole numbers	***2-digit numbers, without regrouping***	***2-digit numbers, with regrouping***	***Explains a renaming algorithm for addition***
7.6	Subtracts multidigit whole numbers	***2-digit numbers, without regrouping***	***2-digit numbers, with regrouping***	***Explains a renaming algorithm for subtraction***

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Competencies for Measurement

Does the student understand and use these big ideas?

- Attributes of objects can be measured or compared
- Uniform units allow attributes of objects to be quantified
- Measures can be determined by repeating a unit or using a tool

Table 10. Framework for Concept Area 8: Measurement

Concept Area 8: Measurement

		Performance level		
Learning goal		A	B	C
8.1	Makes comparisons based on measurable attributes	Compares length directly	Compares length indirectly	***Orders 3 objects by length***
8.2	Measures length	By laying multiple length units end-to-end	By iterating a single length unit	***By using a ruler***
8.3	Understands units		***Recognizes the need for equal-sized units***	***Estimates change in measurement based on change in unit***

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Competencies for Geometry

Does the student understand and use these big ideas?

- Geometric shapes can be identified by their attributes, regardless of their orientation
- Geometric shapes can be composed and decomposed into other shapes

Table 11. Framework for Concept Area 9: Geometry

Concept Area 9: Geometry

	Performance level		
Learning goal	A	B	C
9.1 Identifies quadrilterals in standard orientation	Squares	***Normal proportion rectangles***	***Rectangles with exaggerated aspect ratio***
9.2 Identifies triangles in standard orientation	***Equilateral triangles in point-up orientation***	***Nonequilateral triangles in point-up orientation***	
9.3 Identifies geometric figures in nonstandard orientation	***Squares***	Rectangles	Triangles
9.4 Identifies components and properties of shapes			***Identifies shapes based on their properties***
9.5 Composes geometric figures	Simple frames with distinct outlines	No frame provided	By substituting a combination of smaller shapes for a larger shape

Content limits shown in ***bold and italic*** indicate that the item is in the diagnostic assessment but not the grade-level assessment.

Kindergarten learning goals

Table 12 lists learning goals and content limits for kindergarten assessment items. The goal numbers and performance levels correspond to the location of each item in one of the nine concept areas.

Table 12. Learning Goals for Kindergarten Assessment Items

Concept area	Item no.	Goal no.	Performance level	Learning goal	Content limit
Verbal Counting	K1	1.1	A	Counts by ones	To 10
	K2	1.1	B	Counts by ones	To 20
	K3	1.1	C	Counts by ones	To at least 40
	K4	1.2	A	Counts forward from variable starting points	Start value in teens
	K5	1.2	B	Counts forward from variable starting points	From 10
Counting Objects	K6	2.1	A	Counts objects in a given collection	To 10
	K7	2.1	B	Counts objects in a given collection	To 20
	K8	2.2	A	Produces a collection of a specified size	To 10
	K9	2.2	B	Produces a collection of a specified size	To 20
	K10	2.4	A	Writes the numeral to represent a quantity	To 10
	K11	2.4	B	Writes the numeral to represent a quantity	To 20
Measurement	K12	8.1	A	Makes comparisons based on measurable attributes	Compares length directly
	K13	8.1	B	Makes comparisons based on measurable attributes	Compares length indirectly
Adding to and Taking From in Contexts	K14	3.1	A	Solves context problems of the type join, result unknown (JRU)	Totals 2 to 6
	K15	3.1	B	Solves context problems of the type join, result unknown (JRU)	Totals 7 to 10
	K16	3.2	A	Solves context problems of the type separate, result unknown (SRU)	Totals 2 to 6
	K17	3.2	B	Solves context problems of the type separate, result unknown (SRU)	Totals 7 to 10
	K18	3.3	B	Solves context problems of the type part-part-whole, whole unknown (PPW-WU)	Totals 7 to 10
	K19	3.4	B	Solves context problems of the type part-part-whole, part unknown (PPW-PU)	Totals 7 to 10
	K20	3.5	B	Solves context problems of the type separate, change unknown (SCU)	Totals 7 to 10
	K21	3.6	B	Solves context problems of the type compare, difference unknown (CDU)	Totals 7 to 10

Concept area	Item no.	Goal no.	Performance level	Learning goal	Content limit
Comparing and Ordering Numbers	K22	4.1	A	Compares sets or numbers	Sets to 10
	K23	4.1	B	Compares sets or numbers	Sets vs. numbers to 20
	K24	4.2	A	Orders 3 or more numbers	3 numbers less than 10
Fluency With Number Combinations	K25	5.1	A	Composes and decomposes numbers	Constructs partners, with objects, totals to 5
	K26	5.1	B	Composes and decomposes numbers	Constructs partners, with objects, totals to 10
	K27	5.3	A	Knows addition combinations for doubles	Totals 2 to 5
	K28	5.4	A	Knows addition combinations for near doubles	Totals 2 to 5
	K29	5.8	A	Knows subtraction combinations for doubles	Totals 2 to 5
Place Value	K30	7.2	A	Translates among place value models, count words, numerals	Teens
	K31	7.3	A	Reads and writes multidigit numbers meaningfully	To 20
Geometry	K32	9.1	A	Identifies quadrilaterals in standard orientation	Squares
	K33	9.3	B	Identifies geometric figures in non-standard orientation	Rectangles
	K34	9.3	C	Identifies geometric figures in non-standard orientation	Triangles
	K35	9.5	A	Composes geometric figures	Simple frames with distinct outlines

First-grade learning goals

Table 13 lists learning goals and content limits for the first-grade assessment items. The goal numbers and performance levels correspond to the location of each item in one of the nine concept areas.

Table 13. Learning Goals for First-Grade Assessment Items

Concept area	Item no.	Goal no.	Performance level	Learning goal	Content limit
Verbal Counting	F1	1.2	C	Counts forward from variable starting points	Start value above 50
Comparing and Ordering Numbers	F2	4.2	B	Orders 3 or more numbers	4 numbers to 20
	F3	4.2	C	Orders 3 or more numbers	5 numbers to 100
Adding to and Taking From in Contexts	F4	3.1	C	Solves context problems of the type join, result unknown (JRU)	Totals 11 to 18
	F5	3.2	C	Solves context problems of the type separate, result unknown (SRU)	Totals 11 to 18
	F6	3.3	C	Solves context problems of the type part-part-whole, whole unknown (PPW-WU)	Totals 11 to 18
	F7	3.4	C	Solves context problems of the type part-part-whole, part unknown (PPW-PU)	Totals 11 to 18
	F8	3.5	C	Solves context problems of the type separate, change unknown (SCU)	Totals 11 to 18
	F9	3.6	C	Solves context problems of the type compare, difference unknown (CDU)	Totals 11 to 18
	F10	3.7	C	Solves context problems of the type separate, start unknown (SSU)	Totals 11 to 18
	F11	3.8	C	Solves context problems of the type join, start unknown (JSU)	Totals 11 to 18
Measurement	F12	8.2	A	Measures length	By laying multiple length units end-to-end
	F13	8.2	B	Measures length	By iterating a single length unit
Counting Objects	F14	2.4	C	Writes the numeral to represent a quantity	To 100
Geometry	F15	9.5	B	Composes geometric figures	No frame provided
	F16	9.5	C	Composes geometric figures	By substituting a combination of smaller shapes for a larger shape

Concept area	Item no.	Goal no.	Performance level	Learning goal	Content limit
Fluency With Number Combinations	F17	5.5	B	Knows addition combinations based on 10	Totals equal to 10
	F18	5.6	B	Knows other addition combinations	Totals 6 to 9
	F19	5.9	B	Knows subtraction combinations for near doubles	Totals 6 to 9
	F20	5.10	B	Knows subtraction combinations based on 10	Totals equal to 10
	F21	5.11	B	Knows other subtraction combinations	Totals 6 to 9
Properties and Symbols	F22	6.1	B	Translates between word problems and number sentences	SCU, totals 11 to 18
	F23	6.1	C	Translates between word problems and number sentences	JSU, totals 11 to 18
	F24	6.2	A	Identifies the connection between add/sub and counting forward/ backward	Connects adding to counting on
	F25	6.4	B	Recognizes and uses properties of addition	Commutative property
Place Value	F26	7.2	B	Translates among place value models, count words, numerals	2-digit numbers
	F27	7.3	B	Reads and writes multidigit numbers meaningfully	2-digit numbers
	F28	7.4	B	Decomposes a larger unit into smaller units by place value	2-digit numbers

Diagnostic assessment learning goals

Table 14 lists learning goals and content limits for the diagnostic assessment items. The goal numbers and performance levels correspond to the location of each item in one of the nine concept areas.

Items on the diagnostic assessment that also appear on a grade-level assessment include the grade-level item number next to the diagnostic assessment item number. Items that appear only on the diagnostic assessment and not on a grade-level assessment show the grade-level cell shaded green. Kindergarten items are purple (K ■), first-grade items are orange (F ■), and above first-grade items are blue (above F ■).

Table 14. Learning Goals for Diagnostic Assessment Items

Concept area	Diagnostic item no.	Grade-level item no.	Goal no.	Performance level	Learning goal	Content limit
Verbal Counting	D1	K1	1.1	A	Counts by ones	To 10
	D2	K2	1.1	B	Counts by ones	To 20
	D3	K3	1.1	C	Counts by ones	To at least 40
	D4	K4	1.2	A	Counts forward from variable starting points	Start value less than 10
	D5	K5	1.2	B	Counts forward from variable starting points	Start value in teens
	D6	F1	1.2	C	Counts forward from variable starting points	Start value above 50
	D7		1.3	A	Counts backward by ones from variable starting points	From 10
	D8		1.3	B	Counts backward by ones from variable starting points	From 20
	D9		1.3	C	Counts backward by ones from variable starting points	Start value above 50 (across decades)
	D10		1.4	A	Uses skip counting	By tens to 100
	D11		1.4	B	Uses skip counting	By fives to 55
	D12		1.4	C	Uses skip counting	By twos to 24
Counting Objects	D13	K6	2.1	A	Counts objects in a given collection	To 10
	D14	K7	2.1	B	Counts objects in a given collection	To 20
	D15		2.1	C	Counts objects in a given collection	To 100 with objects grouped in tens
	D16	K8	2.2	A	Produces a collection of a specified size	To 10
	D17	K9	2.2	B	Produces a collection of a specified size	To 20
	D18		2.2	C	Produces a collection of a specified size	To 100
	D19		2.3	A	Recognizes collections arranged in patterns without counting	To 4
	D20		2.3	B	Recognizes collections arranged in patterns without counting	To 6

Concept area	Diagnostic item no.	Grade-level item no.	Goal no.	Performance level	Learning goal	Content limit
Counting Objects (cont'd.)	D21		2.3	C	Recognizes collections arranged in patterns without counting	To 10
	D22	K10	2.4	A	Writes the numeral to represent a quantity	To 10
	D23	K11	2.4	B	Writes the numeral to represent a quantity	To 20
	D24	F14	2.4	C	Writes the numeral to represent a quantity	To 100
Adding to and Taking From in Contexts	D25	K14	3.1	A	Solves context problems of the type join, result unknown (JRU)	Totals 2 to 6
	D26	K15	3.1	B	Solves context problems of the type join, result unknown (JRU)	Totals 7 to 10
	D27	F4	3.1	C	Solves context problems of the type join, result unknown (JRU)	Totals 11 to 18
	D28	K16	3.2	A	Solves context problems of the type separate, result unknown (SRU)	Totals 2 to 6
	D29	K17	3.2	B	Solves context problems of the type separate, result unknown (SRU)	Totals 7 to 10
	D30	F5	3.2	C	Solves context problems of the type separate, result unknown (SRU)	Totals 11 to 18
	D31		3.3	A	Solves context problems of the type part-part-whole, whole unknown (PPW-WU)	Totals 2 to 6
	D32	K18	3.3	B	Solves context problems of the type part-part-whole, whole unknown (PPW-WU)	Totals 7 to 10
	D33	F6	3.3	C	Solves context problems of the type part-part-whole, whole unknown (PPW-WU)	Totals 11 to 18
	D34		3.4	A	Solves context problems of the type part-part-whole, part unknown (PPW-PU)	Totals 2 to 6
	D35	K19	3.4	B	Solves context problems of the type part-part-whole, part unknown (PPW-PU)	Totals 7 to 10
	D36	F7	3.4	C	Solves context problems of the type part-part-whole, part unknown (PPW-PU)	Totals 11 to 18
	D37		3.5	A	Solves context problems of the type separate, change unknown (SCU)	Totals 2 to 6
	D38	K20	3.5	B	Solves context problems of the type separate, change unknown (SCU)	Totals 7 to 10
	D39	F8	3.5	C	Solves context problems of the type separate, change unknown (SCU)	Totals 11 to 18

Concept area	Diagnostic item no.	Grade-level item no.	Goal no.	Performance level	Learning goal	Content limit
Adding to and Taking From in Contexts (cont'd.)	D40		3.6	A	Solves context problems of the type compare, difference unknown (CDU)	Totals 2 to 6
	D41	K21	3.6	B	Solves context problems of the type compare, difference unknown (CDU)	Totals 7 to 10
	D42	F9	3.6	C	Solves context problems of the type compare, difference unknown (CDU)	Totals 11 to 18
	D43		3.7	A	Solves context problems of the type separate, start unknown (SSU)	Totals 2 to 6
	D44		3.7	B	Solves context problems of the type separate, start unknown (SSU)	Totals 7 to 10
	D45	F10	3.7	C	Solves context problems of the type separate, start unknown (SSU)	Totals 11 to 18
	D46		3.8	A	Solves context problems of the type join, start unknown (JSU)	Totals 2 to 6
	D47		3.8	B	Solves context problems of the type join, start unknown (JSU)	Totals 7 to 10
	D48	F11	3.8	C	Solves context problems of the type join, start unknown (JSU)	Totals 11 to 18
Comparing and Ordering Numbers	D49	K22	4.1	A	Compares sets or numbers	Sets to 10
	D50	K23	4.1	B	Compares sets or numbers	Sets vs. numbers to 20
	D51		4.1	C	Compares sets or numbers	Numbers to 100
	D52	K24	4.2	A	Orders 3 or more numbers	3 numbers less than 10
	D53	F2	4.2	B	Orders 3 or more numbers	4 numbers to 20
	D54	F3	4.2	C	Orders 3 or more numbers	5 numbers to 100
	D55		4.3	A	Represents numbers on the number line	To 10, with grid marks
	D56		4.3	B	Represents numbers on the number line	To 20, with grid marks
	D57		4.3	C	Represents numbers on the number line	To 100, with only decade grid marks
	D58		4.4	A	Identifies ordinal position	To 10th
	D59		4.4	B	Identifies ordinal position	To 30th
	D60		4.4	C	Identifies ordinal position	Reads ordinal terms (words) through 9th and uses them
	D61		4.5	A	Determines how many more or less	By comparing sets (to 10)

Concept area	Diagnostic item no.	Grade-level item no.	Goal no.	Performance level	Learning goal	Content limit
Comparing and Ordering Numbers (cont'd.)	D62		4.5	B	Determines how many more or less	By counting on (numbers to 20, differences within 5)
	D63		4.5	C	Determines how many more or less	By adding or subtracting, to 100
Fluency With Number Combinations	D64	K25	5.1	A	Composes and decomposes numbers	Constructs partners, with objects, totals to 5
	D65	K26	5.1	B	Composes and decomposes numbers	Constructs partners, with objects, totals to 10
	D66		5.2	A	Knows "plus 1" is next counting word	Totals 2 to 5
	D67		5.2	B	Knows "plus 1" is next counting word	Totals 6 to 9
	D68	K27	5.3	A	Knows addition combinations for doubles	Totals 2 to 5
	D69		5.3	B	Knows addition combinations for doubles	Totals 6 to 9
	D70		5.3	C	Knows addition combinations for doubles	Totals 11 to 18
	D71	K28	5.4	A	Knows addition combinations for near doubles	Totals 2 to 5
	D72		5.4	B	Knows addition combinations for near doubles	Totals 6 to 9
	D73		5.4	C	Knows addition combinations for near doubles	Totals 11 to 18
	D74	F17	5.5	B	Knows addition combinations based on 10	Totals equal to 10
	D75		5.5	C	Knows addition combinations based on 10	Combinations with 9
	D76	F18	5.6	B	Knows other addition combinations	Totals 6 to 9
	D77		5.6	C	Knows other addition combinations	Totals 11 to 18
	D78		5.7	A	Knows "minus 1" is previous counting word	Totals 2 to 5
	D79		5.7	B	Knows "minus 1" is previous counting word	Totals 6 to 9
	D80	K29	5.8	A	Knows subtraction combinations for doubles	Totals 2 to 5
	D81		5.8	B	Knows subtraction combinations for doubles	Totals 6 to 9
	D82		5.8	C	Knows subtraction combinations for doubles	Totals 11 to 18

Concept area	Diagnostic item no.	Grade-level item no.	Goal no.	Performance level	Learning goal	Content limit
Fluency With Number Combinations (cont'd.)	D83		5.9	A	Knows subtraction combinations for near doubles	Totals 2 to 5
	D84	F19	5.9	B	Knows subtraction combinations for near doubles	Totals 6 to 9
	D85		5.9	C	Knows subtraction combinations for near doubles	Totals 11 to 18
	D86	F20	5.10	B	Knows subtraction combinations based on 10	Totals equal to 10
	D87		5.10	C	Knows subtraction combinations based on 10	Combinations with 9
	D88	F21	5.11	B	Knows other subtraction combinations	Totals 6 to 9
	D89		5.11	C	Knows other subtraction combinations	Totals 11 to 18
Properties and Symbols	D90		6.1	A	Translates between word problems and number sentences	JRU, totals 2 to 9
	D91	F22	6.1	B	Translates between word problems and number sentences	SCU, totals 11 to 18
	D92	F23	6.1	C	Translates between word problems and number sentences	JSU, totals 11 to 18
	D93	F24	6.2	A	Identifies the connection between add/sub and counting forward/backward	Connects adding to counting on
	D94		6.2	B	Identifies the connection between add/sub and counting forward/backward	Connects subtracting to counting back
	D95		6.3	A	Compares numbers using symbols	Using =
	D96		6.3	B	Compares numbers using symbols	Using <, >
	D97		6.3	C	Compares numbers using symbols	Produces symbol
	D98	F25	6.4	B	Recognizes and uses properties of addition	Commutative property
	D99		6.4	C	Recognizes and uses properties of addition	Associative property to add 3 single-digit numbers
	D100		6.5	A	Recognizes addition subtraction complement and inverse principle	Inverse principle, single-digit numbers in a context
	D101		6.5	B	Recognizes addition subtraction complement and inverse principle	Complement principle in a context
	D102		6.5	C	Recognizes addition subtraction complement and inverse principle	Inverse principle in symbols

Concept area	Diagnostic item no.	Grade-level item no.	Goal no.	Performance level	Learning goal	Content limit
Place Value	D103		7.1	A	Recognizes base-ten equivalents	10 ones = 1 ten
	D104		7.1	B	Recognizes base-ten equivalents	10 tens = 100 ones = 1 hundred
	D105		7.1	C	Recognizes base-ten equivalents	10 hundreds = 1,000
	D106	K30	7.2	A	Translates among place value models, count words, numerals	Teens
	D107	F26	7.2	B	Translates among place value models, count words, numerals	2-digit numbers
	D108		7.2	C	Translates among place value models, count words, numerals	3-digit numbers
	D109	K31	7.3	A	Reads and writes multidigit numbers meaningfully	To 20
	D110	F27	7.3	B	Reads and writes multidigit numbers meaningfully	2-digit numbers
	D111		7.3	C	Reads and writes multidigit numbers meaningfully	3-digit numbers
	D112		7.4	A	Decomposes a larger unit into smaller units by place value	To 30
	D113	F28	7.4	B	Decomposes a larger unit into smaller units by place value	2-digit numbers
	D114		7.4	C	Decomposes a larger unit into smaller units by place value	3-digit numbers
	D115		7.5	A	Adds multidigit whole numbers	2-digit numbers, without regrouping
	D116		7.5	B	Adds multidigit whole numbers	2-digit numbers, with regrouping
	D117		7.5	C	Adds multidigit whole numbers	Explains a renaming algorithm for addition
	D118		7.6	A	Subtracts multidigit whole numbers	2-digit numbers, without regrouping
	D119		7.6	B	Subtracts multidigit whole numbers	2-digit numbers, with regrouping
	D120		7.6	C	Subtracts multidigit whole numbers	Explains a renaming algorithm for subtraction
Measurement	D121	K12	8.1	A	Makes comparisons based on measurable attributes	Compares length directly
	D122	K13	8.1	B	Makes comparisons based on measurable attributes	Compares length indirectly
	D123		8.1	C	Makes comparisons based on measurable attributes	Orders 3 objects by length

Concept area	Diagnostic item no.	Grade-level item no.	Goal no.	Performance level	Learning goal	Content limit
Measurement (cont'd.)	D124	F12	8.2	A	Measures length	By laying multiple length units end-to-end
	D125	F13	8.2	B	Measures length	By iterating a single length unit
	D126		8.2	C	Measures length	By using a ruler
	D127		8.3	B	Understands units	Recognizes the need for equal-sized units
	D128		8.3	C	Understands units	Estimates change in measurement based on change in unit
Geometry	D129	K32	9.1	A	Identifies quadrilaterals in standard orientation	Squares
	D130		9.1	B	Identifies quadrilaterals in standard orientation	Normal proportion rectangles
	D131		9.1	C	Identifies quadrilaterals in standard orientation	Rectangles with exaggerated aspect ratio
	D132		9.2	A	Identifies triangles in standard orientation	Equilateral triangles in point-up orientation
	D133		9.2	B	Identifies triangles in standard orientation	Nonequilateral triangles in point-up orientation
	D134		9.3	A	Identifies geometric figures in nonstandard orientation	Squares
	D135	K33	9.3	B	Identifies geometric figures in nonstandard orientation	Rectangles
	D136	K34	9.3	C	Identifies geometric figures in nonstandard orientation	Triangles
	D137		9.4	C	Identifies components and properties of shapes	Identifies shapes based on their properties
	D138	K35	9.5	A	Composes geometric figures	Simple frames with distinct outlines
	D139	F15	9.5	B	Composes geometric figures	No frame provided
	D140	F16	9.5	C	Composes geometric figures	By substituting a combination of smaller shapes for a larger shape

Directions for Using Grade-Level Assessments

Gaining insight through student interviews

The kindergarten and first-grade assessments in Assessing Mathematical Understanding are administered through a personal interview. During the interview, students use their knowledge and apply their preferred strategies to determine the answer for each item without the assistance of a peer or the adult administering the assessment. Through this interview process, students show what they know about each mathematical concept.

> "Interviews and observations are more appropriate assessment techniques than group tests, which often do not yield complete data. Early assessment should be used to gain information for teaching and for potential early interventions, rather than for sorting children."—NCTM, 2000, p. 75

The person administering the assessment interview observes and records the strategies and explanations the student uses while solving each item. This type of assessment provides rich data about student knowledge that cannot be obtained in a group setting. Teachers who have used similar one-on-one interviews to assess students report that "the data from interviews were revealing of student mathematical understanding and development in a way that would not be possible without that special opportunity for one-to-one interaction" (Clarke, 2000, as cited in State of Victoria, Department of Education, Employment and Training, 2001, p. 8).

> "[Assessing Mathematical Understanding] is extremely helpful in assessing each student's knowledge in the different concept areas. You find out more in an individual math assessment than you can in group or whole class instruction."—First-grade teacher and field tester, Aberdeen School District, Washington

Each grade-level assessment provides direction for movement through the items based on student responses to each item. The image to the right is found in the lower right-hand section of each teacher page and tells which item to administer next if a student answers incorrectly.

In this way, administration of the assessment is individualized for each student. When administered again, the assessment can begin with the items a student missed, answered only partially correct, or answered without fluency in the previous administration. It is not necessary to readminister the entire battery of items. The administrator can omit items which the student demonstrated proficiency on previously, thereby reducing the amount of time required to complete the assessment.

Assessing Mathematical Understanding's grade-level assessments are color-coded for easy reference. Every element related to kindergarten is purple and first grade is orange.

Instructions for interviews

Preparation

1. **Collect the materials** necessary for the assessment.
2. **Set up a space** that is free from distractions and allows the teacher or other test administrator and student to sit comfortably face-to-face with the test booklet open on the table between them. There should be sufficient workspace for the student to lay out manipulatives and to write.
3. **Bring one student at a time** to the interview location.
4. **Read the introductory script.**
   a. Say, "Today I am going to ask you some number questions. Do you like number questions?"
   b. Say, "It's OK to say, 'I don't know,' or 'Let's move on,' for any question."
   c. Say, "I will read a problem over again, if you ask me to."
   d. Say, "You may use any of the objects on the table to help you think about the question."
   e. Say, "Are you ready to begin? OK, let's get started." (Or wait if the student has a question.)

Administration and Scoring

5. **Read each item as printed and elaborate, if necessary.** The goal is for the student to be able to show what he or she knows.
   a. You may paraphrase or repeat anything in the assessment.
   b. You may offer manipulatives shown on each page.
   c. Students may point (rather than speak) to indicate an answer when appropriate.
   d. If a student does not know his or her colors or is unfamiliar with a vocabulary word, you may clarify.
   e. There is no time limit for responses (except as indicated in the assessment).
   f. Units are not required for correct answers. For example, "5" and "5 dogs" are both correct.
6. **Give neutral feedback** that does not indicate whether the student has answered correctly or incorrectly. Maintain a neutral expression. Reinforce students' good effort. The following are some sample responses you might use:
   a. "Thank you."
   b. "I see just what you did."
   c. "Good work!"
   d. "Was that a hard/easy problem?"
   e. "Nice job!"
   f. "Shall we go on to the next one?"
7. **Record student responses** to each item and mark the student record using the codes indicated on page 39.
8. **Follow the "moving through the assessment" directions.** In the lower right portion of each teacher's page there are instructions telling whether to advance to the next question or skip to a later question if the student answers incorrectly.

After Each Assessment

9. Complete the learning profile on the student record.
10. Compute a cumulative score using the point values indicated.

Student records

The individual student record is designed to be a cumulative record of the student's growth in proficiency over time. Results for each assessment are collected on the same record, which provides evidence of student progress over the course of the year.

Student Responses. One side of the student record provides space to note the student's responses to each item. Table 15 provides a sample First-Grade Student Record. In addition to writing the student's answers to each question, use the following codes to indicate the type of answer provided:

Response Codes

Ⓒ correct answer given quickly with confidence (3 points)
C correct answer (2 points)
P partially correct answer (1 point)
X incorrect answer, no response to the problem or "I don't know" (0 points)
S skipped based on skip criteria (0 points)

Additional codes can be used to indicate details about student responses:

? indicates student hesitated or was puzzled by the task
DM indicates student used direct modeling to solve (fingers, objects)
CO indicates student used a counting strategy (counting on, counting back)
NR indicates student used numerical reasoning to solve (known fact, mental calculation, number relationships)
SC indicates student self-corrected (initially gave an incorrect answer, then changed to correct answer)

Table 15. Sample First-Grade Student Record

Concept area: Verbal Counting				
Item	Correct response	Student's response Date 9/30/10	Student's response Date	Student's response Date
1	67, 68, 69, 70, 71, 72, 73, 74, 75	C		

Concept area: Comparing and Ordering Numbers				
Item	Correct response	Student's response Date 9/30/10	Student's response Date 1/5/11	Student's response Date
2	7, 10, 14, 16	P 7, 10, 16, 14	C	
3	17, 70, 78, 80, 87	S	P 17, 70, 80, 78, 87	

Concept area: Adding to and Taking From in Contexts				
Item	Correct response	Student's response Date 9/30/10	Student's response Date 1/5/11	Student's Response Date
4	13 carrots	C DM used cubes		
5	5 cookies	C cubes		
6	17 pennies	C 8 double +1 NR		
7	8 girls	C SC		
8	4 pencils	C CO		
9	7 peach trees	C DM		
10	11 birds	X	C CO	
11	8 candies	X	C CO	

Concept area: Measurement				
Item	Correct response	Student's response Date	Student's response Date 1/5/11	Student's response Date
12	Approximately 8 (depends on size of paper clip)		C SC	
13	Approximately 5 (depends on the size of paper clip)		X	

Concept area: Counting Objects				
Item	Correct response	Student's response Date	Student's response Date 1/5/11	Student's response Date
14	Uses 10s and 1s and writes "53"		X ?	

Learning Profile. After interviewing a student during a grade-level assessment, complete the learning profile on the student record. Table 16 provides a sample First-Grade Learning Profile. The teacher may fill this in to show the items in each concept area where the student responded correctly with confidence (3-point responses) or took some time to derive the correct answer (2-point responses). During successive administrations of the assessment, marking additional cells provides a visual profile of growth in mathematical proficiency throughout the course of the year.

- For items scored Ⓒ, correct with confidence, shade in the entire cell.
- For items scored *c,* correct, draw an x in the cell.
- For items scored *p,* partially correct, draw a diagonal line through the cell.
- For items scored *x* or *s,* incorrect or skipped, leave the cell blank.

Cumulative Scores. Following the administration of each assessment, teachers may compute a score for that student using the assessment scoring directions. Table 16 provides a sample first-grade cumulative score after two assessment dates. Scores are determined using a scale that awards three points for a correct answer given with confidence, two points for a correct answer, and one point for a partially correct answer. Changes in the overall score with each successive administration provide one way to document students' progress.

- Write the student's cumulative score for each concept area on the date the assessment is administered.

Table 16. Sample First-Grade Student Record: Learning Profile and Cumulative Score

Learning Profile									
Concept area	Verbal Counting	Comparing and Ordering Numbers	Adding to and Taking From in Contexts	Measure-ment	Counting Objects	Geometry	Fluency With Number Combina-tions	Properties and Symbols	Place Value
Item	1	2	4	12	14	15	17	22	26
		3	5	13		16	18	23	27
			6				19	24	28
			7				20	25	
			8				21		
			9						
			10						
			11						
Cumulative Scores									
Dates									
9/30/10	3	1	13						
1/5/11		4	17	2	3				
Max. score	3	6	24	6	3	6	15	12	9

Instructions for interviews

Preparation

1. **Collect the materials** necessary for the assessment.
2. **Set up a space** that is free from distractions and allows the teacher or other test administrator and student to sit comfortably face-to-face with the test booklet open on the table between them. There should be sufficient workspace for the student to lay out manipulatives and to write.
3. **Bring one student at a time** to the interview location.
4. **Read the introductory script.**
   a. Say, "Today I am going to ask you some number questions. Do you like number questions?"
   b. Say, "It's OK to say, 'I don't know,' or 'Let's move on,' for any question."
   c. Say, "I will read a problem over again, if you ask me to."
   d. Say, "You may use any of the objects on the table to help you think about the question."
   e. Say, "Are you ready to begin? OK, let's get started." (Or wait if the student has a question.)

Administration and Scoring

5. **Read each item as printed and elaborate, if necessary.** The goal is for the student to be able to show what he or she knows.
   a. You may paraphrase or repeat anything in the assessment.
   b. You may offer manipulatives shown on each page.
   c. Students may point (rather than speak) to indicate an answer when appropriate.
   d. If a student does not know his or her colors or is unfamiliar with a vocabulary word, you may clarify.
   e. There is no time limit for responses (except as indicated in the assessment).
   f. Units are not required for correct answers. For example, "5" and "5 dogs" are both correct.
6. **Give neutral feedback** that does not indicate whether the student has answered correctly or incorrectly. Maintain a neutral expression. Reinforce students' good effort. The following are some sample responses you might use:
   a. "Thank you."
   b. "I see just what you did."
   c. "Good work!"
   d. "Was that a hard/easy problem?"
   e. "Nice job!"
   f. "Shall we go on to the next one?"
7. **Record student responses** to each item and mark the student record using the codes indicated on page 38.
8. **Follow the "moving through the assessment" directions.** In the lower right portion of each teacher's page there are instructions telling whether to advance to the next question or skip to a later question if the student answers incorrectly.

After Each Assessment

9. Complete the learning profile on the student record.
10. Compute a cumulative score using the point values indicated.

Class record

The class record provided with the assessment allows the teacher to document and see at a glance the progress of the entire class. Assessment results compiled on a class record can help the teacher make appropriate decisions about the formation of flexible learning groups focused on particular concepts and skills. See Table 17 for a sample filled in by a teacher.

The method for completing the class record sheet is similar to that of the Learning Profile:

- For items scored Ⓒ, correct with confidence, shade in the entire cell.
- For items scored ***C***, correct, draw an x in the cell.
- For items scored ***P***, partially correct, draw a diagonal line through the cell.
- For items scored ***X*** or ***S***, incorrect or skipped, leave the cell blank.

> "The assessment helps me know where students are on a particular concept or skill, which allows me to compare abilities and make flexible student groups."
> —Teacher and field-test participant, Helena School District, Montana

Table 17. Sample Class Record

	Student names	1 Sophia	2 Charlotte	3 Ava	4 Olivia	5 Emily	6 Liam	7 Noah	8 Jackson	9 Ethan	10 Mason	11 Lily	12 Chloe	13 Isabella	14 Emma	15 Abigail	16 Oliver	17 Logan	18 Jacob	19 Ben	20 Caleb	21 Audrey	22 Grace	23 James	24 Alex	25 Owen
	Item / Learning Goal																									
	Concept area: Verbal Counting																									
1	**1.1A** Counts by ones (to 10)					X											X									
2	**1.1B** Counts by ones (to 20)					/					/	/					/									
3	**1.1C** Counts by ones (to at least 40)				/				/	/	/	/	/			/			/	/						
4	**1.2A** Counts forward from variable starting points (start value less than 10)						/																/			/
5	**1.2B** Counts forward from variable starting points (start value in teens)	X	/					X						/	/						/			X		
	Concept area: Counting Objects																									
6	**2.1A** Counts objects in a given collection (to 10)					X											X									
7	**2.1B** Counts objects in a given collection (to 20)					/	/						/							/	/					X

Scheduling interviews

The grade-level assessments are designed to be used with all students. When administered two or three times during the year, they provide data that can track the progress students are making in the nine concept areas. Each grade-level assessment can be administered in its entirety in about 20 to 30 minutes per student. Each grade-level assessment can also be administered in three sections requiring about 6 to 10 minutes per student. Intermediate stopping points are clearly marked in the assessment booklet and on the student records.

Many teachers will find the following administration schedule manageable. Naturally, this schedule can be modified to fit the teacher's curriculum.

Table 18. School-Year Calendar

Month	Kindergarten	First grade
September/October	Assess Section 1 (13 items): • Verbal Counting • Counting Objects • Measurement	Assess Section 1 (11 items): • Fluency With Number Combinations • Place Value • Geometry
January/February	Reassess any previously missed items Assess Section 2 (11 items) • Adding to and Taking From in Contexts • Comparing and Ordering Numbers	Reassess any previously missed items Assess Section 2 (10 items) • Measurement • Counting Objects • Geometry • Fluency With Number Combinations
May/June	Reassess any previously missed items Assess Section 3 (11 items) • Fluency With Number Combinations • Place Value • Geometry	Reassess any previously missed items Assess Section 3 (7 items) • Properties and Symbols • Place Value

"The assessment gives a good picture of where kids are developmentally and shows growth when used two or three times per year."

—Second-grade teacher and field test participant, Aberdeen School District, Washington

"Using this assessment three times in the year is a great way to gather data that show mathematical growth."

—First-grade teacher and field test participant, Helena School District, Montana

Directions for Using the Diagnostic Assessment

Tracking student growth

The purpose of the diagnostic assessment is to ascertain individual student's strengths, weaknesses, knowledge, and skills in particular concept areas so that the teacher can adjust instruction or provide appropriate interventions. Based on the results of the grade-level assessment and classroom observations, teachers may identify some individual students for whom additional, in-depth assessment data are desired. These might be students who score substantially above or below the rest of the students in the class. They may be students identified by a prior teacher as struggling with mathematics. For each of these students, a teacher can choose particular concept areas of concern and administer the bank of items in those sections of the diagnostic assessment.

Most of the items in each section of the diagnostic assessment are provided at three levels of performance for each learning goal to help the teacher pinpoint a student's level of success in each concept area. This allows the teacher to pose increasingly more or less difficult tasks for each learning goal to determine a student's level of proficiency. The diagnostic assessment is organized by concept area and contains color-coded items at the kindergarten and first-grade levels that use the same color scheme as the grade-level assessment. Every element related to kindergarten is purple. Every element related to first grade is orange. A third color, blue, is included in the diagnostic assessment to indicate learning goals that are either beyond kindergarten and first grade or concepts that are not explicitly stated in the Common Core State Standards. Inclusion of the blue-coded items is based on research about learning trajectories and can provide the teacher with a more complete picture of a child's understanding of mathematics.

This inclusive assessment provides the teacher with detailed information about what to observe in terms of student strategies, approaches, and potential misconceptions. The "For Further Diagnosis" section gives the teacher information about what to observe in the student's performance. It also offers probing questions that may elicit more detail about student understanding of mathematical concepts.

> "The diagnostic assessment pinpoints which learning goals need attention. It provides information about what to observe along with splendid probing questions to deepen teacher to student communication."
>
> —Mathematics specialist and reviewer, Kent School District, Washington

Diagnostic assessment records

The Diagnostic Assessment Student Record contains the items for each concept area and provides ample space for recording detailed information that may include observations about children's strategies and reasoning for each item. It also indicates the item number, learning goal with content limit for each item, and the grade level associated with that item. Completed records point out areas of strength and potential holes in a student's mathematical knowledge. See Table 19 for a sample completed by a teacher.

Table 19. Sample Diagnostic Assessment Record

Concept area: Verbal Counting				
Item	Learning goal	Correct response	Code	Student's response
D1 (K1)	1.1A Counts by ones (to 10)	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	(C)	
D2 (K2)	1.1B Counts by ones (to 20)	11, 12, 13, 14, 15, 16, 17, 18, 19, 20	(C)	
D3 (K3)	1.1C Counts by ones (to at least 40)	21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40 41, 42, 43, 44		20, 28, 30, 35, 40 Student seemed satisfied with answer
D4 (K4)	1.2A Counts forward from variable starting points (start value less than 10)	4, 5, 6, 7, 8, 9, 10, 11	(C)	
D5 (K5)	1.2B Counts forward from variable starting points (start value in teens)	16, 17, 18, 19, 20, 21, 22	C	Had to start over. First said 16, 17, 18, 20, 21, 22
D6 (F1)	1.2C Counts forward from variable starting points (start value above 50)	67, 68, 69, 70, 71, 72, 73, 74, 75	C	Took 2 tries SC (self-corrected)
D7	1.3A Counts backwards by ones from variable starting points (from 10)	10, 9, 8, 7, 6, 5, 4, 3, 2, 1	C	Used both hands and put fingers down when counting
D8	1.3B Counts backwards by ones from variable starting points (from 20)	20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1	P	Skipped 17, but did not notice
D9	1.3C Counts backwards by ones from variable starting points (start value above 50—across decades)	92, 91, 90, 89, 88, 87, 86, 85	P	Slowly started 92, 91, 90, 80 – SC Started over 92, 91, 90, 87, 86, 85
D10	1.4A Uses skip counting (by tens to 100)	10, 20, 30, 40, 50, 60, 70, 80, 90, 100	(C)	Enjoyed this!

The diagnostic assessment contains items for kindergarten, first grade, and above. These allow the teacher to move easily between items at, above, or below the grade level of the student in order to pinpoint the level of success for each learning goal. Kindergarten students who are successful with all the items for their grade level can be assessed with items at the next grade level. First-grade students who struggle with items for their grade level can be assessed with items at the previous grade level. Items beyond first grade are also included to give a comprehensive picture of a child's mathematical understanding.

> "[The diagnostic assessment] is very comprehensive … I like the way you can see the continuum from kindergarten to first grade. This gives real understanding of where kids are and where they are headed."
> —Teacher and reviewer, Portland Public Schools, Oregon

Instructions for interviews

Preparation

1. **Collect the materials** necessary for the assessment.
2. **Set up a space** that is free from distractions and allows the teacher or other assessment administrator and student to sit comfortably face-to-face with the test booklet open on the table between them. There should be sufficient workspace for the student to lay out manipulatives and to write.
3. **Bring one student at a time** to the interview location.
4. **Read the introductory script.**
   a. Say, "Today I am going to ask you some more number questions. I am really interested in how you think about these problems so I might ask you to explain your thinking."
   b. Say, "It's OK to say, 'I don't know,' or 'Let's move on,' for any question."
   c. Say, "I will read a problem over again, if you ask me to."
   d. Say, "You may use any of the objects on the table to help you think about the question."
   e. Say, "Are you ready to begin? OK, let's get started." (Or wait if the student has a question.)

Administration and Data Collection

5. **Read each item as printed and elaborate, if necessary.** The goal is for the student to be able to show what he or she knows.
   a. You may paraphrase or repeat anything in the assessment.
   b. You may offer manipulatives shown on each page.
   c. Students may point (rather than speak) to indicate an answer when appropriate.
   d. If a student does not know his or her colors or is unfamiliar with a vocabulary word, you may clarify.
   e. There is no time limit for responses (except as indicated in the assessment).
   f. Units are not required for correct answers. For example, "5" and "5 dogs" are both correct.
6. **Use the "For Further Diagnosis" suggestions** found on the teacher's page for each item to gain additional information about a student's thinking about that item. Additional probing questions and prompts that can be used include:
   a. "How did you know that?"
   b. "Tell me out loud what you did."
   c. "What were you thinking?"
   d. "How did you figure that out?"
   e. "Can you show me another way to do that?"

7. **Record detailed notes about student responses on the record.** The most important information to collect during the diagnostic assessment is detailed descriptions of the student response and strategies. If desired, the same codes from the grade-level assessment can be used to capture some information; however, no score will be computed for the diagnostic sections:

   Ⓒ correct answer given quickly with confidence
   C correct answer
   P partially correct answer
   X incorrect answer, no response to the problem, or "I don't know"

   Other codes can be used to indicate details about student responses:

   ? indicates student hesitated or was puzzled by the task
   DM indicates student used direct modeling to solve (fingers, objects)
   CO indicates student used a counting strategy (counting on, counting back)
   NR indicates student used numerical reasoning to solve (known fact, mental calculation, number relationships)
   SC indicates student self-corrected (initially gave an incorrect answer, then changed to correct answer)

The student record for the diagnostic assessment provides room for the teacher to take extensive notes about the student's strategy, what he and she says, and any additional prompts or questions used by the teacher. By assessing students in this individualized way, teachers gain rich information about what the student understands, as well as the strategies and reasoning he or she uses. In this way the diagnostic assessment pinpoints which learning goals need attention so that the teacher can adjust instruction to meet the needs of that child.

Collecting additional observational data

Assessing Mathematical Understanding should serve as one data source providing teachers with a gauge of student progress toward meeting important mathematical learning goals. Its purpose is not to label students, but to help teachers to adjust instruction in response to student progress. The results from Assessing Mathematical Understanding can be used as a starting point for teachers to make additional observations about what students know and can do during the day-to-day learning in the classroom.

In addition to Assessing Mathematical Understanding, teachers should use varied and authentic assessments, including observation, documentation of children's talk, informal interviews, collection of student work over time, and the use of open-ended questions. Such instructional strategies illuminate children's thinking, giving the teacher a full picture of students' mathematical strengths and needs (Lindquist & Joyner, 2004).

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Appendix A

Pilot and field tests

The initial process to develop the items in Assessing Mathematical Understanding involved numerous steps. The assessment developers began with a thorough review of the relevant research and literature, including examination of existing assessment items from other countries. They also examined the NCTM Focal Points to identify the targeted concept areas for each grade. Using this background information, the team drafted a set of potential grade-level items for pilot tests.

The development of Assessing Mathematical Understanding included three phases of testing. During the initial phase, potential items for each learning goal at each level were piloted with kindergarten, first-, and second-graders to find out how students responded to the prompts, materials, and format of the assessment. These tests were administered to students at schools in Aberdeen School District and Camas School District in Washington and Lebanon Community Schools in Oregon. Student responses to the pilot-test items were evaluated by the item writers and informed several revisions of the individual items. The pilot tests ultimately resulted in a bank of draft items representing the 140 cells in the assessment framework.

After the items were compiled and sequenced into three grade-level assessments, the development team conducted a second set of field tests to gather data on assessment procedures and on the psychometric properties of the items for each grade-level assessment. Developers conducted these tests at two schools in the Aberdeen School District and in four Lebanon Community Schools. In each location, the assessment was administered to students by the assessment developers, classroom teachers, paraeducators, and other support personnel at the school. Trainers from Education Northwest, formerly Northwest Regional Educational Laboratory, provided school-based staff with an orientation and training session prior to their use of the assessment with students. The field tests provided data from 219 students who were representative of each district's student population.

After the field tests, the 23 participating teachers, paraeducators, and other support personnel completed a brief questionnaire to provide feedback on implementation issues and to note specific problems with the assessment or individual items. These responses were reviewed by the item developers and used to modify items and elements of the assessment format prior to the final field tests.

The final phase of testing occurred in January 2009 and included teachers and students from the same two schools in Aberdeen, two of the original four schools in Lebanon, and 11 schools in the Helena School District in Montana. In each location, the assessment was administered to students by classroom teachers, paraeducators, and other school support personnel who received training from Education Northwest staff. In Lebanon and Aberdeen, the assessment was also administered to students by the assessment developers. These field tests yielded data from 409 students who were representative of each district's student population.

After the tests, the 72 participating teachers, paraeducators, and other support personnel completed a questionnaire to provide feedback on implementation issues and to note specific problems with the test or individual items. In addition, six teachers participated in a follow-up focus group session conducted via audioconference. The assessment developers reviewed the responses and modified assessment items and structural elements of the format.

After both sets of field tests, the team conducted statistical analyses on the student data. Item difficulty levels and interitem reliability were analyzed for the three grade-level assessments. Item difficulties were used to ensure each assessment included a balance of easy to difficult items. Interitem correlations were used to identify how well each item correlated with all the other items. Items with low correlations were examined to determine if problems could be identified and corrected. Coefficient alpha, which can range from 0.0 to 1.0, was used to measure the overall level of item consistency for each domain assessment. A final alpha value of more than .90 was obtained for all three grade-level assessments, indicating a high level of internal consistency among test items.

With the adoption of the Common Core State Standards for Mathematics in 2010, Education Northwest mathematics staff members reviewed the assessment items again to determine if they were aligned with the standards. This analysis resulted in several items moving from one grade level to another; in most cases the movement was to a lower grade level. Because of this shift, an insufficient number of items were available for a second-grade assessment.

Some of the grade-level items that did not align with the Common Core State Standards were placed in the diagnostic assessment. The original second-grade items that were not added to a grade-level assessment were retained as diagnostic items. These can be considered reflective of above first grade.

Appendix B

Reproducible records

Kindergarten Assessment Student Record (page 1 of 3)

Student's name_____

School year _____

Teacher's name _____

Response Codes

Use the following codes to categorize the student's response to each item:

- Ⓒ correct answer given quickly with confidence (3 points)
- **C** correct answer (2 points)
- **P** partially correct answer (1 point)
- **X** incorrect answer, no response to the problem or "I don't know" (0 points)
- **S** skipped based on skip criteria (0 points)

Learning Profile and Cumulative Scores

Learning Profile:

- For items scored Ⓒ, correct with confidence, shade in the entire cell.
- For items scored ***C***, correct, draw an x in the cell.
- For items scored ***P***, partially correct, draw a diagonal line through the cell.
- For items scored ***X*** or ***S***, incorrect or skipped, leave the cell blank.

Cumulative Scores:

- Write the student's cumulative score for each concept area on the date the assessment is administered.

Learning Profile								
Concept area	Verbal Counting	Counting Objects	Measure-ment	Adding to and Taking From in Contexts	Comparing and Ordering Numbers	Fluency With Number Combina-tions	Place Value	Geometry
Item	1	6	12	14	22	25	30	32
	2	7	13	15	23	26	31	33
	3	8		16	24	27		34
	4	9		17		28		35
	5	10		18		29		
		11		19				
				20				
				21				
Cumulative Scores								
Dates								
Max. score	15	18	6	24	9	15	6	12

Individual Kindergarten Student Scores (page 2 of 3)

Student's name _____ School year _____

Concept area: Verbal Counting				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
1	1, 2, 3, 4, 5, 6, 7, 8, 9, 10			
2	11, 12, 13, 14, 15, 16, 17, 18, 19, 20			
3	21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44			
4	4, 5, 6, 7, 8, 9, 10, 11			
5	16, 17, 18, 19, 20, 21, 22			

Concept area: Counting Objects				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
6	6 (cubes)			
7	17 (cubes)			
8	Places 9 cubes			
9	Places 16 cubes			
10	Writes "8"			
11	Writes "14"			

Concept area: Measurement				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
12	String			
13	Green and purple lines			

Concept area: Adding to and Taking From in Contexts				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
14	5 (crayons)			
15	9 (apples)			
16	4 (birds)			
17	3 (cars)			
18	9 (people)			
19	2 (yellow shirts)			
20	5 (stickers)			
21	6 (more)			

Individual Kindergarten Student Scores (page 3 of 3)

Student's name _____ School year _____

Concept area: Comparing and Ordering Numbers				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
22	Green (left side)			
23	More pennies in the box			
24	4, 7, 8			

Concept area: Fluency With Number Combinations				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
25	2 (squares)			
26	4 (marbles)			
27	4			
28	5			
29	2			

Concept area: Place Value				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
30	Using base-ten blocks: 1 long and 4 units Using linking cubes: 10 cubes connected and 4 loose cubes			
31	Writes "15"			

Concept area: Geometry				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
32	Blue, green, and purple squares			
33	Yellow and orange rectangles			
34	Purple, green, and orange triangles			
35	Fills space exactly (in any orientation)			

Kindergarten Class Record, Section One

Teacher's name _____ Date _____ Class _____

	Student names	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Item / Learning Goal																									
	Concept area: Verbal Counting																									
1	**1.1A** Counts by ones (to 10)																									
2	**1.1B** Counts by ones (to 20)																									
3	**1.1C** Counts by ones (at least 40)																									
4	**1.2A** Counts forward from variable starting points (start value less than 10)																									
5	**1.2B** Counts forward from variable starting points (start value in teens)																									
	Concept area: Counting Objects																									
6	**2.1A** Counts objects in a given collection (to 10)																									
7	**2.1B** Counts objects in a given collection (to 20)																									
8	**2.2A** Produces a collection of a specified size (to 10)																									
9	**2.2B** Produces a collection of a specified size (to 20)																									
10	**2.4A** Writes the numeral to represent a quantity (to 10)																									
11	**2.4B** Writes the numeral to represent a quantity (to 20)																									
	Concept area: Measurement																									
12	**8.1A** Makes comparisons based on measurable attributes (compares length directly)																									
13	**8.1B** Makes comparisons based on measurable attributes (compares length indirectly)																									

Kindergarten Class Record, Section Two

Teacher's name _____ Date _____ Class _____

	Student names	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Item / Learning Goal																									
	Concept area: Adding to and Taking From in Contexts																									
14	**3.1A** Solves context problems of the type JRU (totals 2 to 6)																									
15	**3.1B** Solves context problems of the type JRU (totals 7 to 10)																									
16	**3.2A** Solves context problems of the type SRU (totals 2 to 6)																									
17	**3.2B** Solves context problems of the type SRU (totals 7 to 10)																									
18	**3.3B** Solves context problems of the type PPW-WU (totals 7 to 10)																									
19	**3.4B** Solves context problems of the type PPW-PU (totals 7 to 10)																									
20	**3.5B** Solves context problems of the type SCU (totals 7 to 10)																									
21	**3.6B** Solves problems of the type CDU (totals 7 to 10)																									
	Concept area: Comparing and Ordering Numbers																									
22	**4.1A** Compares sets or numbers (sets to 10)																									
23	**4.1B** Compares sets or numbers (sets vs. numbers to 20)																									
24	**4.2A** Orders 3 or more numbers (3 numbers less than 10)																									

Kindergarten Class Record, Section Three

Teacher's name _____ Date _____ Class _____

	Student names	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Item / Learning Goal																									
	Concept area: Fluency With Number Combinations																									
25	**5.1A** Composes and decomposes numbers (constructs partners, with objects, totals to 5)																									
26	**5.1B** Composes and decomposes numbers (constructs partners, with objects, totals to 10)																									
27	**5.3A** Knows addition combinations for doubles (totals 2 to 5)																									
28	**5.4A** Knows addition combinations for near doubles (totals 2 to 5)																									
29	**5.8A** Knows subtraction combinations for doubles (totals 2 to 5)																									
	Concept area: Place Value																									
30	**7.2A** Translates among place value models, count words, numerals (teens)																									
31	**7.3A** Reads and writes multidigit numbers meaningfully (to 20)																									
	Concept area: Geometry																									
32	**9.1A** Identifies quadrilaterals in standard orientation (squares)																									
33	**9.3B** Identifies geometric figures in nonstandard orientation (rectangles)																									
34	**9.3C** Identifies quadrilaterals in nonstandard orientation (triangles)																									
35	**9.5A** Composes geometric figures (simple frames with distinct outlines)																									

First-Grade Assessment Student Record (page 1 of 3)

Student's name_____

School year _____

Teacher's name _____

Response Codes

Use the following codes to categorize the student's response to each item:

- ⓒ correct answer given quickly with confidence (3 points)
- **c** correct answer (2 points)
- **p** partially correct answer (1 point)
- **x** incorrect answer, no response to the problem or "I don't know" (0 points)
- **s** skipped based on skip criteria (0 points)

Learning Profile and Cumulative Scores

Learning Profile:

- For items scored ⓒ, correct with confidence, shade in the entire cell.
- For items scored *c,* correct, draw an x in the cell.
- For items scored *p,* partially correct, draw a diagonal line through the cell.
- For items scored *x* or *s,* incorrect or skipped, leave the cell blank.

Cumulative Scores:

- Write the student's cumulative score for each concept area on the date the assessment is administered.

Learning Profile									
Concept area	Verbal Counting	Comparing and Ordering Numbers	Adding to and Taking From in Contexts	Measure-ment	Counting Objects	Geom-etry	Fluency With Num-ber Com-binations	Properties and Symbols	Place Value
Item	1	2	4	12	14	15	17	22	26
		3	5	13		16	18	23	27
			6				19	24	28
			7				20	25	
			8				21		
			9						
			10						
			11						
Cumulative Scores									
Dates									
Max. score	3	6	24	6	3	6	15	12	9

Individual First-Grade Student Scores (page 2 of 3)

Student's name _____ School year _____

Concept area: Verbal Counting				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
1	67, 68, 69, 70, 71, 72, 73, 74, 75			

Concept area: Comparing and Ordering Numbers				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
2	7, 10, 14, 16			
3	17, 70, 78, 80, 87			

Concept area: Adding to and Taking From in Contexts				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
4	13 (carrots)			
5	5 (cookies)			
6	17 (pennies)			
7	8 (girls)			
8	4 (pencils)			
9	7 (peach trees)			
10	11 (birds)			
11	8 (candies)			

Concept area: Measurement				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
12	Approximately 8 (depends on size of paper clip)			
13	Approximately 5 (depends on the size of paper clip)			

Concept area: Counting Objects				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
14	Uses tens and ones and writes "53"			

Individual First-Grade Student Scores (page 3 of 3)

Student's name _____ School year _____

Concept area: Geometry				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
15	Forms a square using 4 right triangles (in any orientation)			
16	Fills space using 6 blocks			

Concept area: Fluency With Number Combinations				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
17	10			
18	6			
19	3			
20	2			
21	3			

Concept area: Properties and Symbols				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
22	15 – ☐ = 6 (middle option)			
23	☐ + 12 = 18 (middle option)			
24	8 + 3 = ☐ (first option)			
25	c) 9 + 5 = 5 + 9			

Concept area: Place Value				
Item	Correct response	Student's response Date _____	Student's response Date _____	Student's response Date _____
26	Uses tens and ones to get 35			
27	Writes "48"			
28	7 (full stacks) with 8 (pennies left over)			

First-Grade Class Record, Section One

Teacher's name _____ Date _____ Class _____

	Student names	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Item / Learning Goal																									
	Concept area: Verbal Counting																									
1	**1.2C** Counts forward from variable starting points (start value above 50)																									
	Concept area: Comparing and Ordering Numbers																									
2	**4.2B** Orders 3 or more numbers (4 numbers to 20)																									
3	**4.2C** Orders 3 or more numbers (5 numbers to 100)																									
	Concept area: Adding to and Taking From in Contexts																									
4	**3.1C** Solves context problems of the type JRU (totals 11 to 18)																									
5	**3.2C** Solves context problems of the type SRU (totals 11 to 18)																									
6	**3.3C** Solves context problems of the type PPW-WU (totals 11 to 18)																									
7	**3.4C** Solves context problems of the type PPW-PU (totals 11 to 18)																									
8	**3.5C** Solves context problems of the type SCU (totals 11 to 18)																									
9	**3.6C** Solves problems of the type CDU (totals 11 to 18)																									
10	**3.7C** Solves context problems of the type SSU (totals 7 to 10)																									
11	**3.8C** Solves context problems of the type JSU (totals 11 to 18)																									

First-Grade Class Record, Section Two

Teacher's name _____ Date _____ Class _____

	Student names	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Item / Learning Goal																									
	Concept area: Measurement																									
12	**8.2A** Measures length (by laying multiple length units end-to-end)																									
13	**8.2B** Measures length (by iterating a single length unit)																									
	Concept area: Counting Objects																									
14	**2.4C** Writes the numeral to represent a quantity (to 100)																									
	Concept area: Geometry																									
15	**9.5B** Composes geometric figures (no frame provided)																									
16	**9.5C** Composes geometric figures (by substituting a combination of smaller shapes for a larger shape)																									
	Concept area: Fluency With Number Combinations																									
17	**5.5B** Knows addition combinations based on 10 (totals equal to 10)																									
18	**5.6B** Knows other addition combinations (totals 6 to 9)																									
19	**5.9B** Knows subtraction combinations near doubles (totals 6 to 9)																									
20	**5.10B** Knows subtraction combinations based on 10 (totals equal to 10)																									
21	**5.11B** Knows other subtraction combinations (totals 6 to 9)																									

First-Grade Class Record, Section Three

Teacher's name _____ Date _____ Class _____

	Student names	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Item Learning Goal																									
	Concept area: Properties and Symbols																									
22	**6.1B** Translates between word problems and number sentences (SCU, totals 11 to 18)																									
23	**6.1C** Translates between word problems and number sentences (JSU, totals 11 to 18)																									
24	**6.2A** Identifies the connection between add/sub and counting forward/backward (connects adding to counting on)																									
25	**6.4B** Recognizes and uses properties of addition (commutative property)																									
	Concept area: Place Value																									
26	**7.2B** Translates among place value models, count words, numerals (2-digit numbers)																									
27	**7.3C** Reads and writes multidigit numbers meaningfully (3-digit numbers)																									
28	**7.4B** Decomposes a larger unit into smaller units by place value (2-digit numbers)																									

Diagnostic Assessment Student Record (concept area 1)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Verbal Counting				
Item	Learning goal	Correct response	Code	Student's response
D1 (K1)	1.1A Counts by ones (to 10)	1, 2, 3, 4, 5, 6, 7, 8, 9, 10		
D2 (K2)	1.1B Counts by ones (to 20)	11, 12, 13, 14, 15, 16, 17, 18, 19, 20		
D3 (K3)	1.1C Counts by ones (to at least 40)	21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40 41, 42, 43, 44		
D4 (K4)	1.2A Counts forward from variable starting points (start value less than 10)	4, 5, 6, 7, 8, 9, 10, 11		
D5 (K5)	1.2B Counts forward from variable starting points (start value in teens)	16, 17, 18, 19, 20, 21, 22		
D6 (F1)	1.2C Counts forward from variable starting points (start value above 50)	67, 68, 69, 70, 71, 72, 73, 74, 75		
D7	1.3A Counts backwards by ones (from 10)	10, 9, 8, 7, 6, 5, 4, 3, 2, 1		
D8	1.3B Counts backwards by ones (from 20)	20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1		
D9	1.3C Counts backwards by ones from variable starting points (start value above 50—across decades)	92, 91, 90, 89, 88, 87, 86, 85		
D10	1.4A Uses skip counting (by tens to 100)	10, 20, 30, 40, 50, 60, 70, 80, 90, 100		
D11	1.4B Uses skip counting (by fives to 55)	5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55		
D12	1.4C Uses skip counting (by twos to 24)	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24		

Diagnostic Assessment Student Record (concept area 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Counting Objects					
Item	Learning goal		Correct response	Code	Student's response
D13 (K6)	2.1A	Counts objects in a given collection (to 10)	6 (cubes)		
D14 (K7)	2.1B	Counts objects in a given collection (to 20)	17 (cubes)		
D15	2.1C	Counts objects in a given collection (to 100 with objects grouped in tens)	Counts by tens and says "40"		
D16 (K8)	2.2A	Produces a collection of a specified size (to 10)	Places 9 cubes		
D17 (K9)	2.2B	Produces a collection of a specified size (to 20)	Places 16 cubes		
D18	2.2C	Produces a collection of a specified size (to 100)	Places 3 ten-sticks and 6 units on the page		
D19	2.3A	Recognizes collections arranged in patterns without counting (to 4)	4 (dots)		
D20	2.3B	Recognizes collections arranged in patterns without counting (to 6)	6 (dots)		
D21	2.3C	Recognizes collections arranged in patterns without counting (to 10)	9 (dots)		
D22 (K10)	2.4A	Writes the numeral to represent a quantity (to 10)	Writes "8"		
D23 (K11)	2.4B	Writes the numeral to represent a quantity (to 20)	Writes "14"		
D24 (F14)	2.4C	Writes the numeral to represent a quantity (to 100)	Uses tens and ones and writes "53"		

Diagnostic Assessment Student Record (concept area 3: page 1 of 3)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Adding To and Taking From in Contexts				
Item	Learning goal	Correct response	Code	Student's response
D25 (K14)	3.1A Solves context problems of the type join, result unknown (JRU, totals 2 to 6)	5 (crayons)		
D26 (K15)	3.1B Solves context problems of the type join, result unknown (JRU, totals 7 to 10)	9 (apples)		
D27 (F4)	3.1C Solves context problems of the type join, result unknown (JRU, totals 11 to 18)	13 (carrots)		
D28 (K16)	3.2A Solves context problems of the type separate, result unknown (SRU, totals 2 to 6)	4 (birds)		
D29 (K17)	3.2B Solves context problems of the type separate, result unknown (SRU, totals 7 to 10)	3 (cars)		
D30 (F5)	3.2C Solves context problems of the type separate, result unknown (SRU, totals 11 to 18)	5 (cookies)		
D31	3.3A Solves context problems of the type part-part-whole, whole unknown (PPW-WU, totals 2 to 6)	4 (reptiles)		
D32 (K18)	3.3B Solves context problems of the type part-part-whole, whole unknown (PPW-WU, totals 7 to 10)	9 (people)		
D33 (F6)	3.3C Solves context problems of the type part-part-whole, whole unknown (PPW-WU, totals 11 to 18)	17 (pennies)		
D34	3.4A Solves context problems of the type part-part-whole, part unknown (PPW-PU, totals 2 to 6)	4 (green apples)		

Diagnostic Assessment Student Record (concept area 3: page 2 of 3)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Adding To and Taking From in Contexts				
Item	Learning goal	Correct response	Code	Student's response
D35 (K19)	3.4B Solves context problems of the type part-part-whole, part unknown (PPW-PU, totals 7 to 10)	2 (yellow shirts)		
D36 (F7)	3.4C Solves context problems of the type part-part-whole, part unknown (PPW-PU, totals 11 to 18)	8 (girls)		
D37	3.5A Solves context problems of the type separate, change unknown (SCU, totals 2 to 6)	3 (toy trucks)		
D38 (K20)	3.5B Solves context problems of the type separate, change unknown (SCU, totals 7 to 10)	5 (stickers)		
D39 (F8)	3.5C Solves context problems of the type separate, change unknown (SCU, totals 11 to 18)	4 (pencils)		
D40	3.6A Solves context problems of the type compare, difference unknown (CDU, totals 2 to 6)	2 (more boys)		
D41 (K21)	3.6B Solves context problems of the type compare, difference unknown (CDU, totals 7 to 10)	6 (more goats)		
D42 (F9)	3.6C Solves context problems of the type compare, difference unknown (CDU, totals 11 to 18)	7 (more peach trees)		
D43	3.7A Solves context problems of the type separate, start unknown (SSU, totals 2 to 6)	5 (brownies)		
D44	3.7B Solves context problems of the type separate, start unknown (SSU, totals 7 to 10)	9 (paintbrushes)		

Diagnostic Assessment Student Record (concept area 3: page 3 of 3)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Adding To and Taking From in Contexts				
Item	Learning goal	Correct response	Code	Student's response
D45 (F10)	3.7C Solves context problems of the type separate, start unknown (SSU, totals 11 to 18)	11 (birds)		
D46	3.8A Solves context problems of the type join, start unknown (JSU, totals 2 to 6)	4 (pencils)		
D47	3.8B Solves context problems of the type join, start unknown (JSU, totals 7 to 10)	3 (trees)		
D48 (F11)	3.8C Solves context problems of the type join, start unknown (JSU, totals 11 to 18)	8 (candies)		

Diagnostic Assessment Student Record (concept area 4: page 1 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Comparing and Ordering Numbers				
Item	Learning goal	Correct response	Code	Student's response
D49 (K22)	4.1A Compares sets or numbers (sets to 10)	Green (left side)		
D50 (K23)	4.1B Compares sets or numbers (sets vs. numbers to 20)	More pennies in the box		
D51	4.1C Compares sets or numbers (numbers to 100)	66		
D52 (K24)	4.2A Orders 3 or more numbers (3 numbers less than 10)	4, 7, 8		
D53 (F2)	4.2B Orders 3 or more numbers (4 numbers to 20)	7, 10, 14, 16		
D54 (F3)	4.2C Orders 3 or more numbers (5 numbers to 100)	17, 70, 78, 80, 87		
D55	4.3A Represents numbers on the number line (to 10, with grid marks)	7		
D56	4.3B Represents numbers on the number line (to 20, with grid marks)	14		
D57	4.3C Represents numbers on the number line (to 100, with only decade grid marks)	57 or 58		
D58	4.4A Identifies ordinal position (to 10th)	Dark blue car (second from end)		
D59	4.4B Identifies ordinal position (to 30th)	Last green bar		
D60	4.4C Identifies ordinal position (reads ordinal terms [words] through 9th and uses them)	Girl with red hair and blue dress		
D61	4.5A Determines how many more or less (by comparing sets, to 10)	3 (more cubes)		
D62	4.5B Determines how many more or less (by counting on, numbers to 20, differences within 5)	4 (more)		

Diagnostic Assessment Student Record (concept area 4: page 2 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Comparing and Ordering Numbers				
Item	Learning goal	Correct response	Code	Student's response
D63	4.5C Determines how many more or less (by adding or subtracting, to 100)	11 (points)		

Diagnostic Assessment Student Record (concept area 5: page 1 of 3)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Fluency With Number Combinations					
Item	Learning goal		Correct response	Code	Student's response
D64 (K25)	5.1A	Composes and decomposes numbers (constructs partners, with objects, totals to 5)	2		
D65 (K26)	5.1B	Composes and decomposes numbers (construct partners, with objects, totals to 10)	4		
D66	5.2A	Knows "plus 1" is next counting word (totals 2 to 5)	4		
D67	5.2B	Knows "plus 1" is next counting word (totals 6 to 9)	7		
D68 (K27)	5.3A	Knows addition combinations for doubles (totals 2 to 5)	4		
D69	5.3B	Knows addition combinations for doubles (totals 6 to 9)	8		
D70	5.3C	Knows addition combinations for doubles (totals 11 to 18)	14		
D71 (K28)	5.4A	Knows addition combinations for near doubles (totals 2 to 5)	5		
D72	5.4B	Knows addition combinations for near doubles (totals 6 to 9)	9		
D73	5.4C	Knows addition combinations for near doubles (totals 11 to 18)	15		
D74 (F17)	5.5B	Knows addition combinations based on 10 (totals equal to 10)	10		

Diagnostic Assessment Student Record (concept area 5: page 2 of 3)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Fluency With Number Combinations					
Item	Learning goal		Correct response	Code	Student's response
D75	5.5C	Knows addition combinations based on 10 (combinations with 9)	14		
D76 (F18)	5.6B	Knows other addition combinations (totals 6 to 9)	6		
D77	5.6C	Knows other addition combinations (totals 11 to 18)	12		
D78	5.7A	Knows "minus 1" is previous counting word (totals 2 to 5)	2		
D79	5.7B	Knows "minus 1" is previous counting word (totals 6 to 9)	7		
D80 (K29)	5.8A	Knows subtraction combinations for doubles (totals 2 to 5)	2		
D81	5.8B	Knows subtraction combinations for doubles (totals 6 to 9)	3		
D82	5.8C	Knows subtraction combinations for doubles (totals 11 to 18)	8		
D83	5.9A	Knows subtraction combinations for near doubles (totals 2 to 5)	2		
D84 (F19)	5.9B	Knows subtraction combinations for near doubles (totals 6 to 9)	3		
D85	5.9C	Knows subtraction combinations for near doubles (totals 11 to 18)	7		
D86 (F20)	5.10B	Knows subtraction combinations based on 10 (totals equal to 10)	2		

Diagnostic Assessment Student Record (concept area 5: page 3 of 3)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Fluency With Number Combinations				
Item	Learning goal	Correct response	Code	Student's response
D87	5.10C Knows subtraction combinations based on 10 (combinations with 9)	6		
D88 (F21)	5.11B Knows other subtraction combinations (totals 6 to 9)	3		
D89	5.11C Knows other subtraction combinations (totals 11 to 18)	5		

Diagnostic Assessment Student Record (concept area 6: page 1 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Properties and Symbols				
Item	Learning goal	Correct response	Code	Student's response
D90	6.1A Translates between word problems and number sentences (JRU, totals 2 to 9)	4 + 3 = ☐ (last option)		
D91 (F22)	6.1B Translates between word problems and number sentences (SCU, totals 11 to 18)	15 – ☐ = 6 (middle option)		
D92 (F23)	6.1C Translates between word problems and number sentences (JSU, totals 11 to 18)	☐ + 12 = 18 (middle option)		
D93 (F24)	6.2A Identifies the connection between add/sub and counting forward/backward (connects adding to counting on)	8 + 3 = ☐ (first option)		
D94	6.2B Identifies the connection between add/sub and counting forward/backward (connects subtracting to counting back)	7 – 2 = ☐ (last option)		
D95	6.3A Compares numbers using symbols (using =)	= (middle option)		
D96	6.3B Compares numbers using symbols (using <, >)	Greater than > (last option)		
D97	6.3C Compares numbers using symbols (produces symbol)	Writes "12 < 50" or "50 > 12"		
D98 (F25)	6.4B Recognizes and uses properties of addition (commutative property)	c) 9 + 5 = 5 + 9		
D99	6.4C Recognizes and uses properties of addition (associative property to add 3 single-digit numbers)	Adds 9 + 1 = 10 first (or another response that accurately uses a rearrangement of the numbers to simplify calculation)		

Diagnostic Assessment Student Record (concept area 6: page 2 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Properties and Symbols				
Item	Learning goal	Correct response	Code	Student's response
D100	6.5A Recognizes addition-subtraction complement and inverse principle (inverse principle, single-digit numbers in a context)	8 (pennies)		
D101	6.5B Recognizes addition-subtraction complement and inverse principle (complement principle in a context)	6 + 5 = 11 and 11 – 6 = 5		
D102	6.5C Recognizes addition-subtraction complement and inverse principle (inverse principle in symbols)	True		

Diagnostic Assessment Student Record (concept area 7: page 1 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Place Value				
Item	Learning goal	Correct response	Code	Student's response
D103	7.1A Recognizes base-ten equivalents (10 ones = 1 ten)	10 (green tickets)		
D104	7.1B Recognizes base-ten equivalents (10 tens = 100 ones = 1 hundred)	10 (boxes)		
D105	7.1C Recognizes base-ten equivalents (10 hundreds = 1,000)	10 (boxes)		
D106 (K30)	7.2A Translates among place value models, count words, numerals (teens)	Uses 1 ten-stick and 4 ones or 1 long and 4 units		
D107 (F26)	7.2B Translates among place value models, count words, numerals (2-digit numbers)	Uses tens and ones to get 35		
D108	7.2C Translates among place value models, count words, numerals (3-digit numbers)	254		
D109 (K31)	7.3A Reads and writes multidigit numbers meaningfully (to 20)	Writes "15"		
D110 (F27)	7.3B Reads and writes multidigit numbers meaningfully (2-digit numbers)	Writes "48"		
D111	7.3C Reads and writes multidigit numbers meaningfully (3-digit numbers)	Writes "574"		
D112	7.4A Decomposes a larger unit into smaller units by place value (to 30)	2 (full pages) with 3 (stickers left over)		
D113 (F28)	7.4B Decomposes a larger unit into smaller units by place value (2-digit numbers)	7 (full stacks) with 8 (pennies left over)		
D114	7.4C Decomposes a larger unit into smaller units by place value (3-digit numbers)	1 more purple ticket and 3 more blue tickets (or 13 more blue tickets)		

Diagnostic Assessment Student Record (concept area 7: page 2 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Place Value					
Item	Learning goal		Correct response	Code	Student's response
D115	7.5A	Adds multidigit whole numbers (2-digit numbers, without regrouping)	Uses a strategy other than count by ones and says "67"		
D116	7.5B	Adds multidigit whole numbers (2-digit numbers with regrouping)	Uses a strategy other than count by ones and says "55"		
D117	7.5C	Adds multidigit whole numbers—free of context (explains a renaming algorithm for addition)	931		
D118	7.6A	Subtracts multidigit whole numbers (2-digit numbers, without regrouping)	Uses a strategy other than count by ones and says "74"		
D119	7.6B	Subtracts multidigit whole numbers (2-digit numbers, with regrouping)	Uses a strategy other than count by ones and says "36"		
D120	7.6C	Subtracts multidigit whole numbers (explains a renaming algorithm for subtraction)	285		

Diagnostic Assessment Student Record (concept area 8)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Measurement				
Item	Learning goal	Correct response	Code	Student's response
D121 (K12)	8.1A Makes comparisons based on measurable attributes (compares length directly)	String		
D122 (K13)	8.1B Makes comparisons based on measurable attributes (compares length indirectly)	Green and purple lines		
D123	8.1C Makes comparisons based on measurable attributes (orders 3 objects by length)	Blue (top is longest)		
D124 (F12)	8.2A Measures length (by laying multiple length units end-to-end)	Approximately 8 (depends on size of paper clip)		
D125 (F13)	8.2B Measures length (by iterating a single length unit)	Approximately 5 (depends on the size of paper clip)		
D126	8.2C Measures length (by using a ruler)	9 (inches)		
D127	8.3B Understands units (recognizes the need for equal-sized units)	No because the paper clips are not all the same size		
D128	8.3C Understands units (estimates change in measurement based on change in unit)	Anywhere between 5 and 7 paper clips		

Diagnostic Assessment Student Record (concept area 9: page 1 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Geometry				
Item	Learning goal	Correct response	Code	Student's response
D129 (K32)	9.1A Identifies quadrilaterals in standard orientation (squares)	Blue, green, and purple squares		
D130	9.1B Identifies quadrilaterals in standard orientation (normal proportion rectangles)	Orange and yellow rectangles		
D131	9.1C Identifies quadrilaterals in standard orientation (rectangles with exaggerated aspect ratio)	Purple, green, and yellow rectangles		
D132	9.2A Identifies triangles in standard orientation (equilateral triangle in point up orientation)	Orange, dark blue, and purple triangles		
D133	9.2B Identifies triangles in standard orientation (nonequilateral triangles in point up orientation)	Blue, purple, green, and orange triangles		
D134	9.3A Identifies geometric figures in nonstandard orientation (squares)	Purple and green squares		
D135 (K33)	9.3B Identifies geometric figures in nonstandard orientation (rectangles)	Yellow and orange rectangles		
D136 (K34)	9.3C Identifies geometric figures in nonstandard orientation (triangles)	Purple, green, and orange triangles		
D137	9.4C Identifies components and properties of shapes (identifies shapes based on their properties)	Right trapezoid (red shape)		
D138 (K35)	9.5A Composes geometric figures (simple frames with distinct outlines)	Fills space exactly (in any orientation)		
D139 (F15)	9.5B Composes geometric figures (no frame provided)	Forms a square using 4 right triangles (in any orientation)		

Diagnostic Assessment Student Record (concept area 9: page 2 of 2)

Name _____ Date _____ Grade _____ Teacher _____

Concept area: Geometry				
Item	Learning goal	Correct response	Code	Student's response
D140 (F16)	9.5C Composes geometric figures (by substituting a combination of smaller shapes for a larger shape)	Fills space using 6 blocks		

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